

## 影像重建 A Course of MRI

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### 本週課程內容

- 脈衝程序圖(Pulse sequences diagram)
- 訊號處理與資料空間 (Signal processing, Data space/K space)

<http://www.ym.edu.tw/~cflu>, Textbook: MRI The Basics, Hashemi et al.

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## 脈衝程序圖

### Pulse Sequences Diagram

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## Pulse Sequences Diagram

- RF pulses
  - $90^\circ$  pulse
  - $180^\circ$  pulse
- Gradients
  - $G_z$ : slice-selective gradient
  - $G_y$ : phase-encoding gradient
  - $G_x$ : frequency-encoding/readout gradient
- Signals
  - Echo

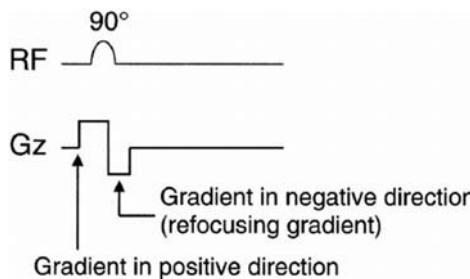
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## Slice-selective gradient $G_z$

- Every time we apply a gradient, we dephase the spins.
- We apply a  $G_z$  gradient to select a slice. But after the slice is selected, we need to reverse the effect of dephasing (refocusing).



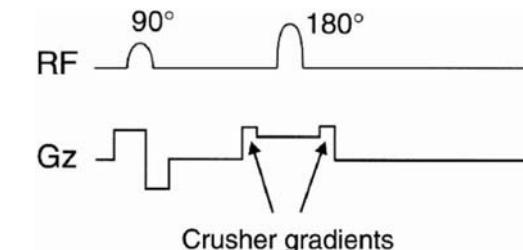
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## Slice-selective gradient $G_z$

- Before and after the 180° pulse, we apply a so-called crusher gradient.
- These crusher gradients are applied to offset the unwanted additional transverse magnetization.



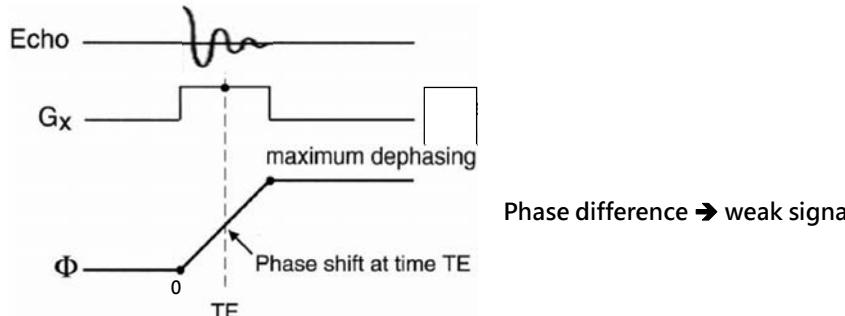
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## Frequency-encoding gradient $G_x$

- Without a proper compensation, the signal intensity will be...
  - decrease at the time in the middle of echo;
  - Maximum dephasing at the time in the end of echo.



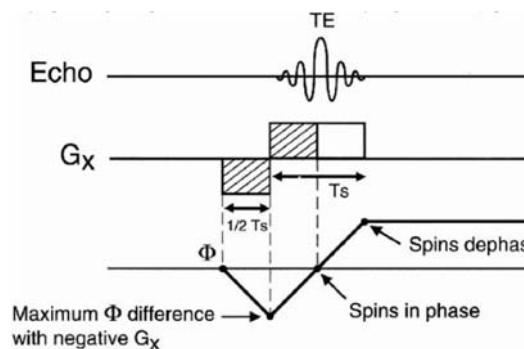
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## Frequency-encoding gradient $G_x$

- We apply a gradient in the negative direction that has an area equal to 1/2 of that of the readout gradient.



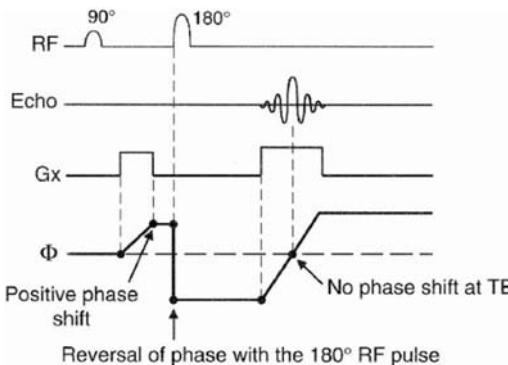
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## Frequency-encoding gradient $G_x$

- Another illustration for the extra  $G_x$  gradient: a positive  $G_x$  gradient before the  $180^\circ$  pulse.
- After the  $180^\circ$  RF pulse, the phase difference will be reversed.



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## Phase-encoding gradient $G_y$

- We need to do a separate phase encode for each row of pixels, that is, 256 rows of pixels costs 256 steps of phase encode.
- We can only perform one phase-encoding step in one TR for a selected slice.
- With each new phase-encoding step, we change the magnetic gradient  $G_y$ .
- $\Delta\theta=360^\circ/\# \text{ of rows}$

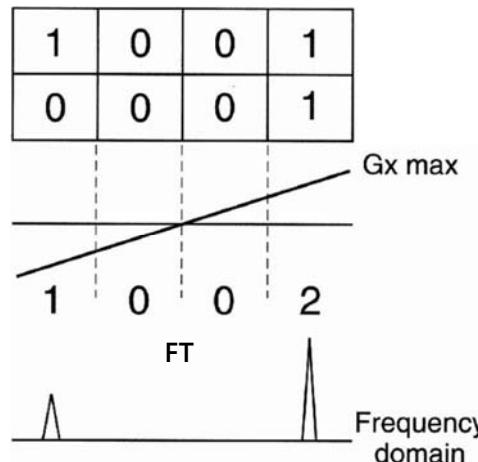
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## An Example: a $2 \times 4$ image

- A frequency encode along x axis can help us to discriminate the difference between columns.



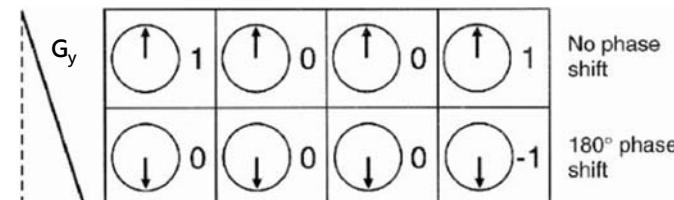
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## An Example: a $2 \times 4$ image

- To discriminate the signals from different rows, we apply a phase encode along y axis.
- $\Delta\theta=360^\circ/2 \text{ rows}=180^\circ$
- We need to apply two different  $G_y$ 
  - Zero phase difference between rows
  - $180^\circ$  phase difference



Multipled by  $\cos(\theta)$   
•  $\cos(0)=1$   
•  $\cos(180)=-1$

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## An Example: a 2 x 4 image

- Zero phase difference

1	0	0	1
0	0	0	1

1	0	0	1
0	0	0	1

- 180° phase difference

1	0	0	1
0	0	0	-1

a	0	0	c
b	0	0	d

- $a+b=1$
- $c+d=2$

- $a-b=1 \rightarrow a = 1, b = 0$
- $c-d=0 \rightarrow c = 1, d = 1$

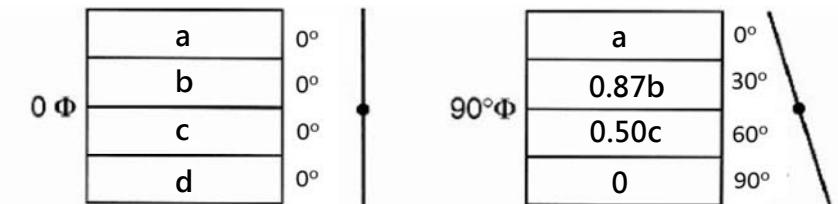
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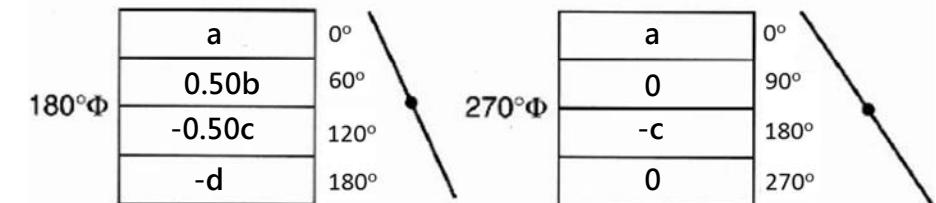
## An Example: a 4 x 1 image

a	0°
b	0°
c	0°
d	0°



a	0°
0.87b	30°
0.50c	60°
0	90°

a	0°
0.50b	60°
-0.50c	120°
-d	180°



a	0°
0	90°
-c	180°
0	270°

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## Time Requirements

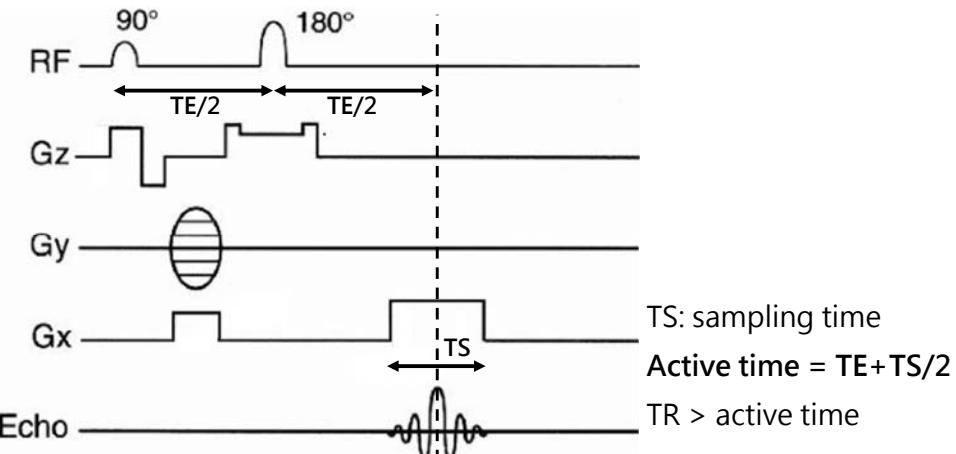
- Each RF pulse (with a  $G_z$  gradient) takes 2~10 msec.
- The phase-encoding step takes 1~5 msec.
- The frequency-encoding step takes about 10msec (4~8 msec at high fields; 16~30 msec at lower fields).

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## A Spin-echo PSD

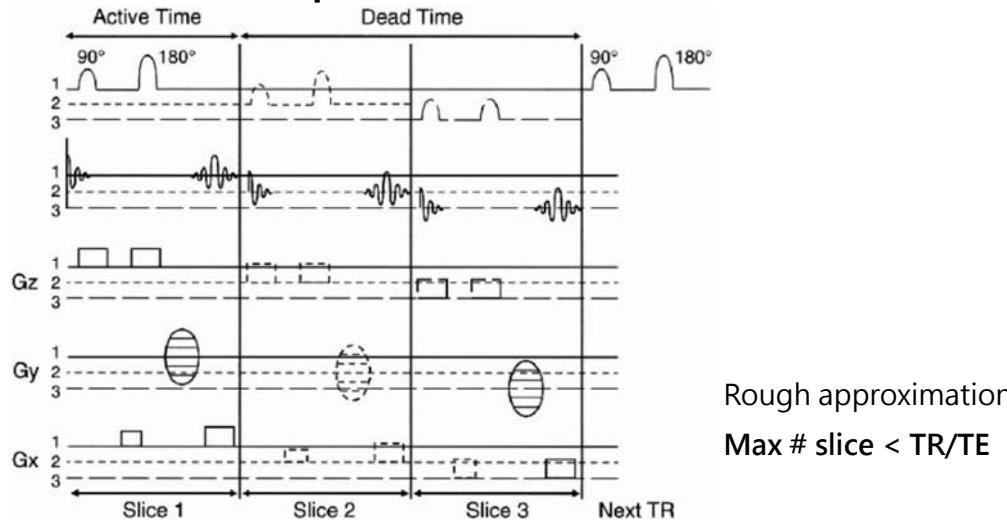


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## Multislice Acquisition in a TR



## 訊號處理與資料空間

Signal Processing, Data Space/K Space

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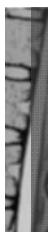
## Data Space

- Each slice has its own data space.
  - Each of received signals (echos) with different phase-encoding gradient fills one line in a set of rows referred to as the data space.
  - Each signal in each row of the data space is the sum of all the signals from individual pixels in the slice.
  - The center of the data space does not represent the center of image.
- 
- +128
- 0
- 127
- Echo from TR#2
- Echo from TR#1

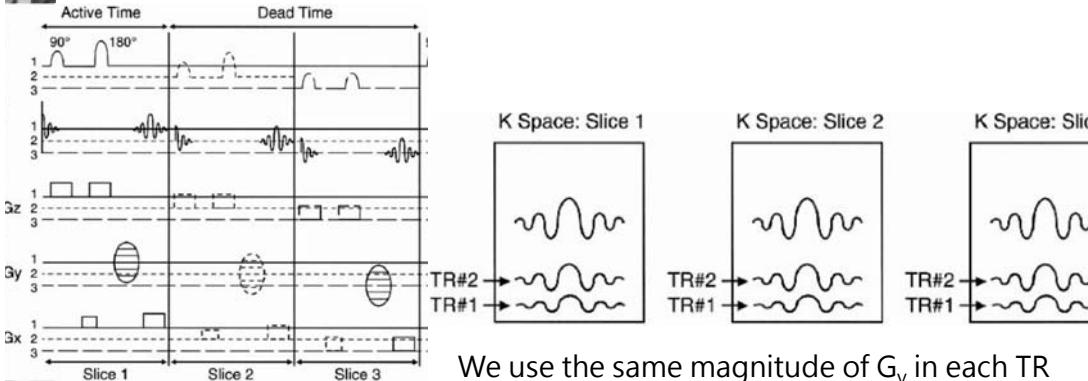
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## Multislice Acquisition in a TR



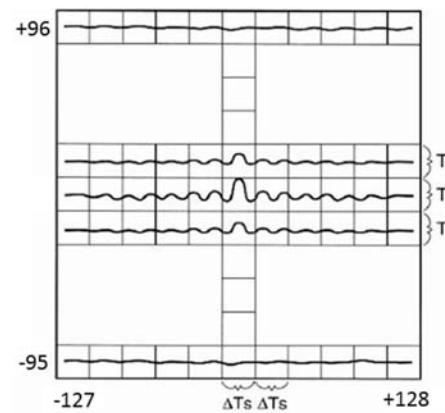
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# K Space

- K space is a digitized (sampled) version of the data space.
- A 256 x 192 k-space matrix
  - The first number represents the different number of frequencies we used.
  - The second number refers to the number of phase encoding steps.



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# Signal-to-Noise Ratio (SNR)

- $\text{SNR} \propto (\text{pixel volume}) \sqrt{\frac{\text{Ny} \times \text{NEX}}{\text{BW}}}$ 
  - BW (receiver bandwidth) =  $1/\Delta Ts$
  - Ny is the number of phase-encoding steps
  - NEX is the number of times we repeat the whole sequence (number of excitations)
  - Pixel volume  $\uparrow$ , spatial resolution  $\downarrow$
  - Ny  $\uparrow$ , spatial resolution  $\uparrow$ , scanning time  $\uparrow$
  - NEX  $\uparrow$ , scanning time  $\uparrow$
  - BW  $\downarrow$ ,  $\Delta Ts \uparrow$ , Ts  $\uparrow$ , TE  $\uparrow$ , T2W  $\uparrow$ , # of slice  $\downarrow$

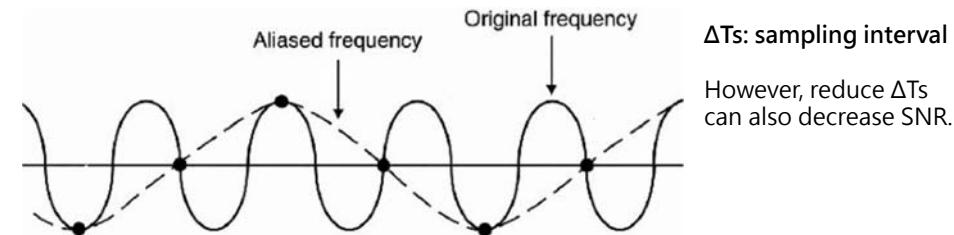
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# Nyquist Sampling theorem

- Analog-to-digital conversion (ADC) is a process by which a time-varying (analog) signal is converted to a digitized form.
- The sampling rate must be at least twice *the maximum signal frequency* to avoid aliasing, i.e.,  $\omega_{\text{sampling}} = 1/\Delta Ts \geq 2\omega_{\text{signal}}$



Oversampling (take more than two samples per cycle) along the frequency-encoding direction can avoid aliasing (wraparound) artifacts.

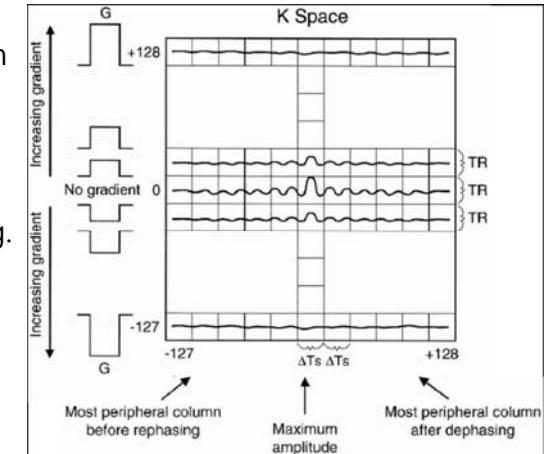
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# Properties of K-Space

- Each of the signals has its maximum signal amplitude in the center column.
- The maximum amplitude occurs in the center row because this line is obtained without additional dephasing.



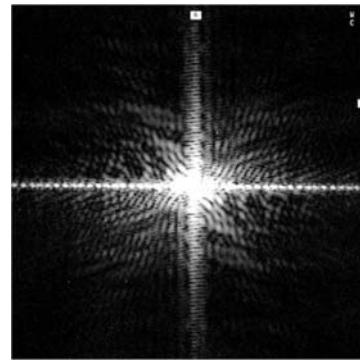
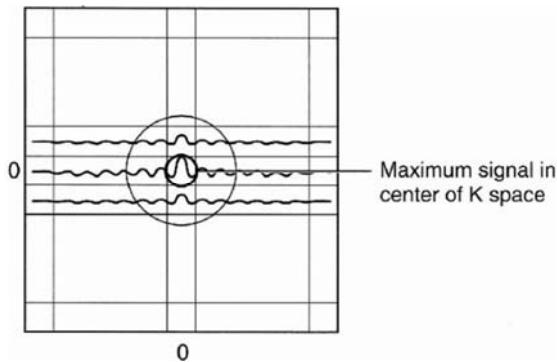
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## Properties of K-Space

- The center point of the data space contains maximum amplitude, i.e., maximum SNR.



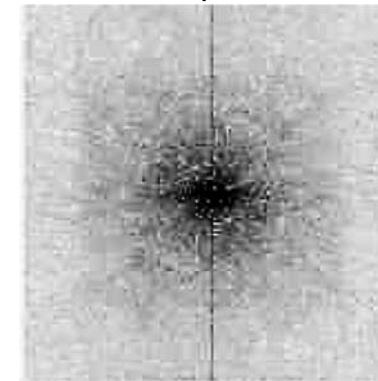
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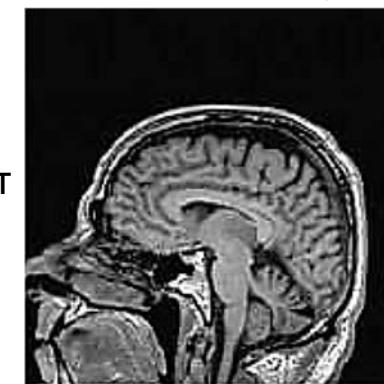
## Image of K-Space

K Space



2D FFT  
→

Reconstructed Image



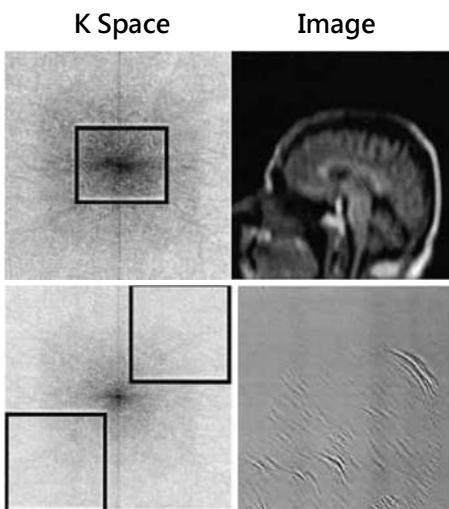
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## Image of K-Space

- The center of k-space contributes to the primary information of image.
- The periphery of k-space provides information regarding fitness of the image and clarity at sharp interfaces



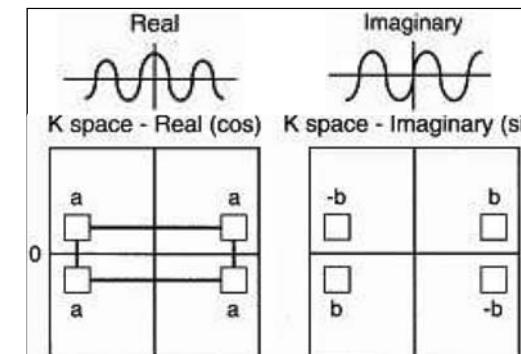
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## Conjugate (Hermitian) Symmetry

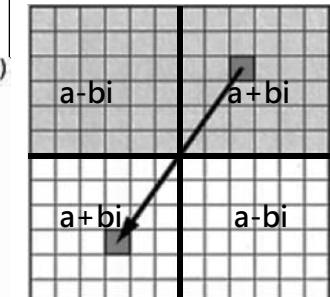
- We preliminarily decompose the signal into its real and imaginary components → a real and an imaginary k space.



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## Magnitude and Phase Image

- Magnitude (modulus) image
  - **Magnitude** =  $\sqrt{a^2 + b^2}$
  - It is what we commonly used in MR imaging.
- Phase (angle) image
  - $\tan\theta = b/a$
  - It is used in cases in which the direction is important.
  - ex: phase contrast MR angiography

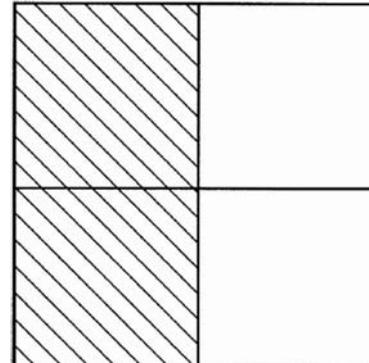
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## Fractional (Partial) Echo

- Only a half of the echo is sampled, and another half is constructed based on the acquired half.
- It allows TE to be shorter.
- The dephasing in the frequency direction is reduced.
- Give better SNR at a given TE when a smaller FOV or thinner slices are selected.
- Gradient echo sequences (FLASH, Fast SPGR)



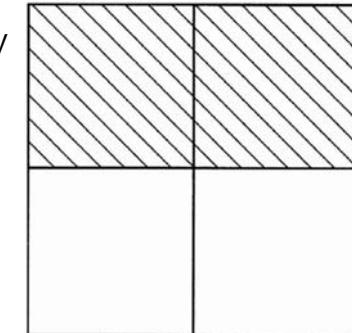
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## Half-Fourier Technique

- We acquire the data from the upper half of k-space and construct the lower part mathematically, thus reducing the scan time.
- The trade-off is a reduced SNR by a factor of  $\sqrt{2}$ .
- Oversampling: we sample half of the phase-encoding steps plus a few lines below the 0 line to compensate the phase errors.



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## Acquisition Time

- The acquisition time depends on
  - TR (the time to do one line of the data space)
  - Ny (the number of phase-encoding steps)
  - NEX (the number of times we repeat the whole sequence to increase SNR)
- $acquisition\ time \propto TR \cdot Ny \cdot NEX$

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# THE END

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