訊號頻譜分析
傅立葉轉換

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請先下載本週上課資料

- **http://www.ym.edu.tw/~cflu**
- 點選左欄〔課程資料〕
- 下載第**7**週上課資料〔**demodata_L6.zip**〕，檔案大小約**2.9MB**

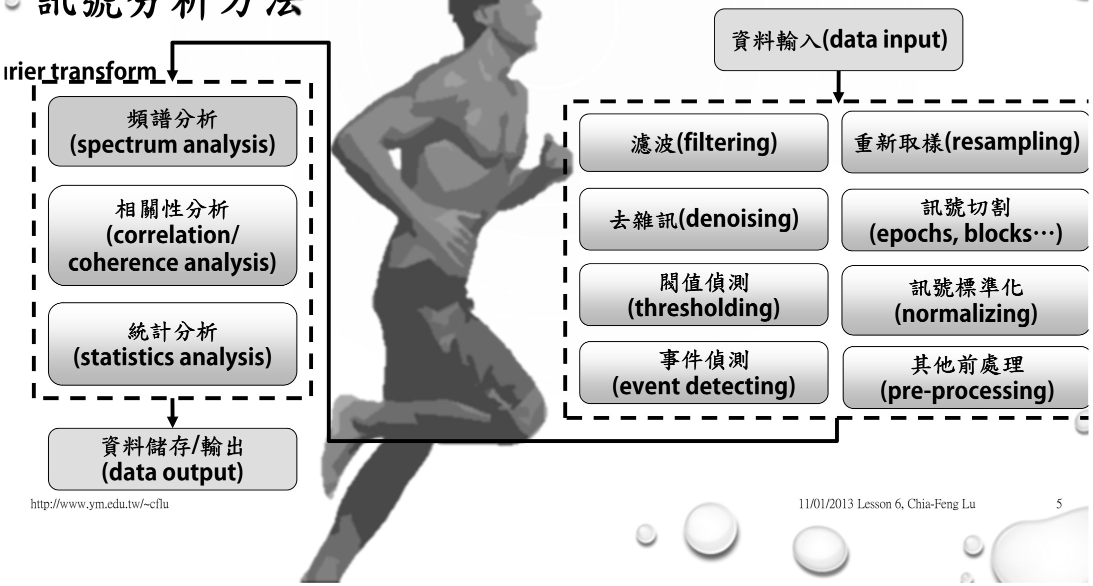
期中報告

- **11/22**口頭報告，
- **11/20**前繳交一頁**A4**構想書（預設分析流程與參考文獻）

本週課程內容

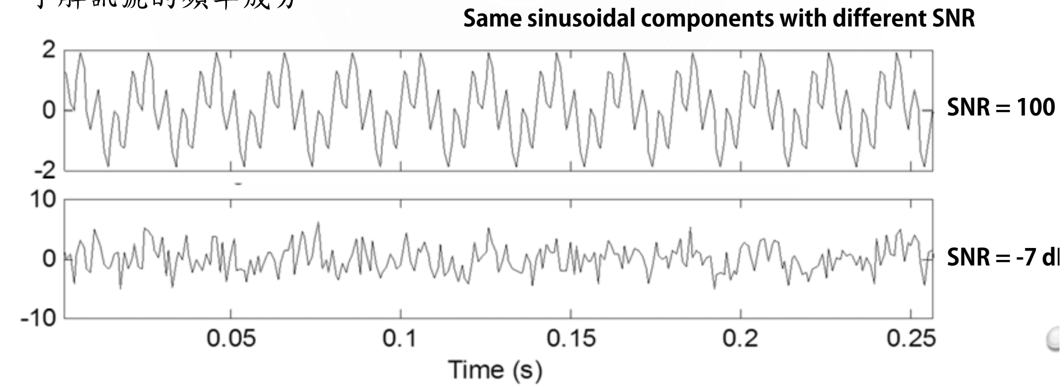
- 傅立葉轉換**(Fourier transform)**
- 頻譜分析**(Spectral analysis)**
- 生理訊號頻譜分析實例

訊號分析方法



頻譜分析(SPECTRAL ANALYSIS)

- 了解訊號的頻率成分



訊雜比(SIGNAL-TO-NOISE RATIO)

- **Root mean square (RMS) of a signal**

$$x_{RMS} = \sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)}$$

- **Signal-to-noise ratio (SNR)**

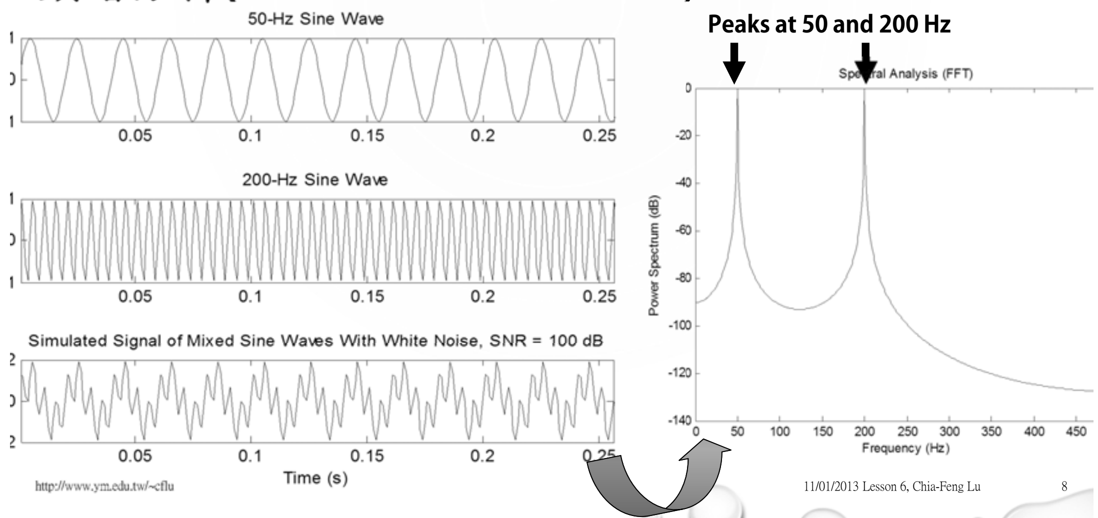
$$SNR_{dB} = 20\log_{10}\left(\frac{signal_{RMS}}{noise_{RMS}}\right)$$

$$SNR_{linear} = 10^{\frac{SNR_{dB}}{20}}$$

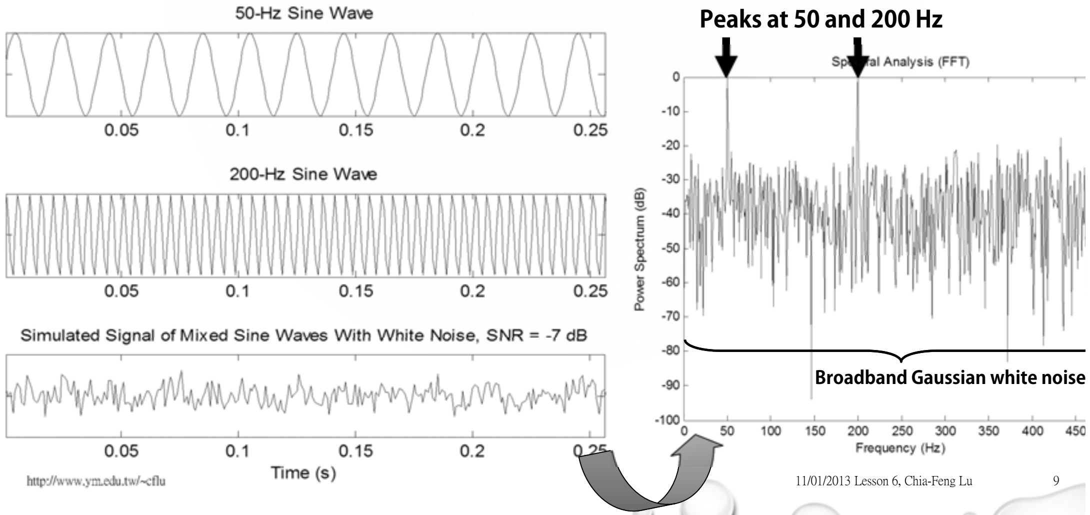
例子：

SNR = -7 dB
即代表訊號強度為雜訊強度的**10^(-7/20)=0.4467**倍

頻譜分析(SPECTRAL ANALYSIS)



頻譜分析(SPECTRAL ANALYSIS)



傅立葉轉換

DISCRETE FOURIER TRANSFORM

傅立葉級數(FOURIER SERIES)

- 任何週期性的波形或訊號，皆可用一系列不同頻率的正(半)弦波加總來表示

$$S(t)=\frac{a_0}{2}+\sum_{n=1}^{N}\left[a_n\cos(2\pi\frac{n}{T}t)+b_n\sin(2\pi\frac{n}{T}t)\right]$$

N: data length (number of data points) of the waveform *S*
T: time length of the waveform *S*

Spectral analysis ➔ a mathematical prism

Lucas V. Barbosa, http://en.wikipedia.org/wiki/File:Fourier_series_and_transform.gif

傅立葉係數(FOURIER COEFFICIENTS)

$$S(t)=\frac{a_0}{2}+\sum_{n=1}^{N}\left[a_n\cos(2\pi\frac{n}{T}t)+b_n\sin(2\pi\frac{n}{T}t)\right]$$

N: data length (number of data points) of the waveform *S*
T: time length of the waveform *S*

$$a_n=\frac{1}{T}\int_0^T S(t)\cos(2\pi\boxed{\frac{n}{T}}t)dt$$

correlate the signal with either cosine or sine, and then avera

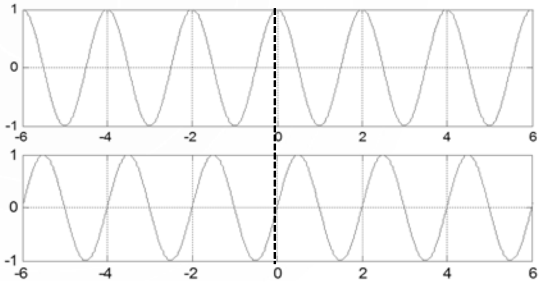
$$b_n=\frac{1}{T}\int_0^T S(t)\sin(2\pi\boxed{\frac{n}{T}}t)dt$$

n creates a family of cosine or sine with harmonic frequency

傳立葉係數(FOURIER COEFFICIENTS)

$$a_n = \frac{1}{T}\int_0^T S(t)\cos(2\pi \frac{n}{T} t)dt$$

$$b_n = \frac{1}{T}\int_0^T S(t)\sin(2\pi \frac{n}{T} t)dt$$



Function Type	Symmetry	Coefficient values
Even	S(t)=S(-t)	b_n=0
Odd	S(t)=-S(-t)	a_n=0
Half-wave	S(t)=S(T-t)	a_n=b_n=0, for n is even

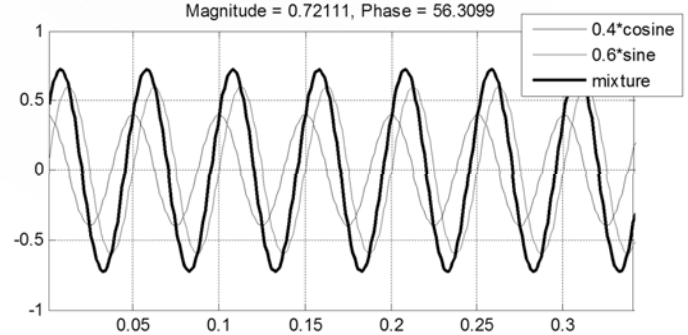
傳立葉係數(FOURIER COEFFICIENTS)

- 也可用一個正弦波函數加上相位變化來表示正弦與半弦波的加成

$$a_n \cos(2\pi \frac{n}{T} t) + b_n \sin(2\pi \frac{n}{T} t) = c_n \sin(2\pi \frac{n}{T} t + \phi_n)$$

$$c_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \arctan\left(\frac{b_n}{a_n}\right)$$



傳立葉轉換(DISCRETE FOURIER TRANSFORM)

- 用複數的表示式來簡化正旋波和半弦波的寫法

$$S(f) = \sum_{n=0}^{N-1} s(n)\cos(2\pi fn/N) - j\sum_{n=0}^{N-1} s(n)\sin(2\pi fn/N)$$

$$= \sum_{n=0}^{N-1} s(n)e^{-j2\pi fn/N}$$

$$|a+bj| = \sqrt{a^2+b^2}$$

$$|a-bj| = \sqrt{a^2+(-b)^2}$$

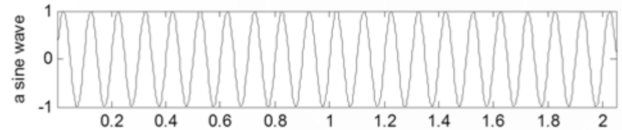
$|S(f)|$

Spectral analysis ➔ a mathematical prism

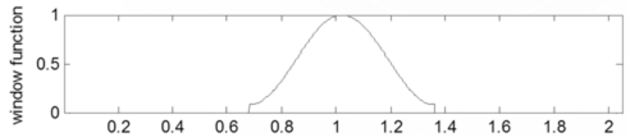
Lucas V. Barbosa, http://en.wikipedia.org/wiki/File:Fourier_series_and_transform.gif

訊號截斷(SIGNAL TRUNCATION)

S

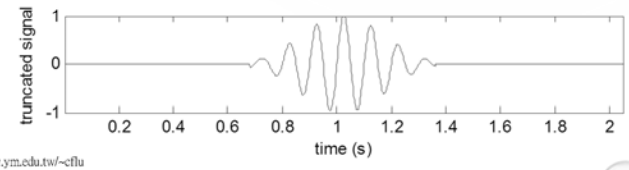


W



Hamming window

S.*W



MATLAB兩個向量的點乘運算

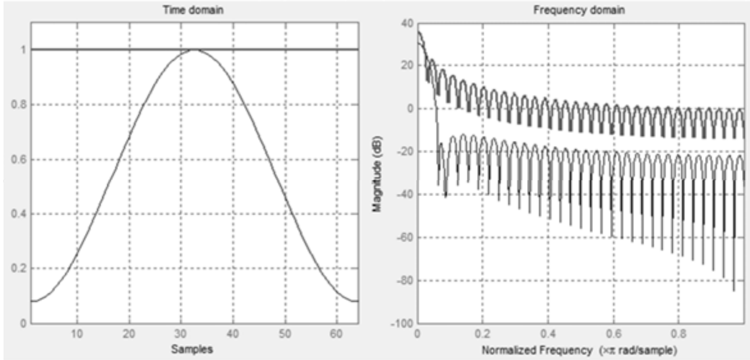
[MATLAB RULE]使用WINDOW FUNCTION

- **help window**
- **W=window(@window_name,length);**

- 請開啟並執行**demodata_L6\TruncateSig_window.m**

有限長度訊號的頻譜假影(ARTIFACTS)

- 頻譜分析會受到截斷訊號時所用的**window function**影響



Rectangular window
- **Mainlobe width (-3dB): 0.02**
- **Sidelobe attenuation: -13.3**

Hamming window
- **Mainlobe width (-3dB): 0.03**
- **Sidelobe attenuation: -42.5**

在**command window**下輸入**wintool**使用工具箱

頻譜分析

SPECTRAL ANALYSIS

功率頻譜(POWER SPECTRUM)

- 時間訊號的能量（**integration of the magnitude of the signal squared**）

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

- 根據**Parseval's theorem**，時間域與頻率域的能量守恆

$$\int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 dt$$

$$PS(f) = |S(f)|^2$$

Lucas V. Barbosa, http://en.wikipedia.org/wiki/File:Fourier_series_and_transform.gif

[MATLAB RULE]快速傳立葉轉換FFT

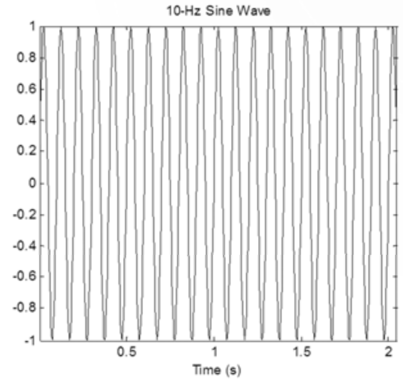
- **help fft**
- **FFT Discrete Fourier transform.**

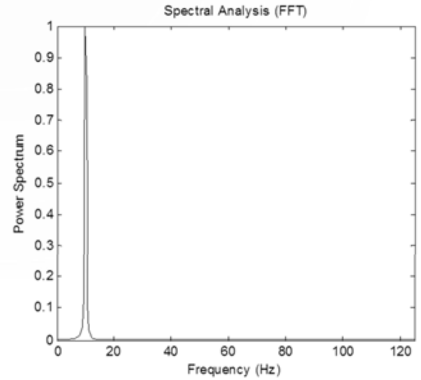
  FFT(X) is the discrete Fourier transform (DFT) of vector X. For matrices, the FFT operation is applied to each column. For N-D arrays, the FFT operation operates on the first non-singleton dimension.

  FFT(X,N) is the N-point FFT, padded with zeros if X has less than N points and truncated if it has more.

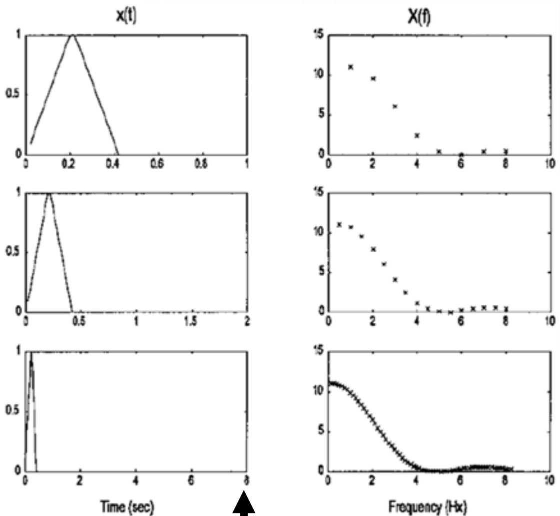
FFT-BASED頻譜分析

- 請開啟並執行**demodata_L6\FFTspectrum.m**





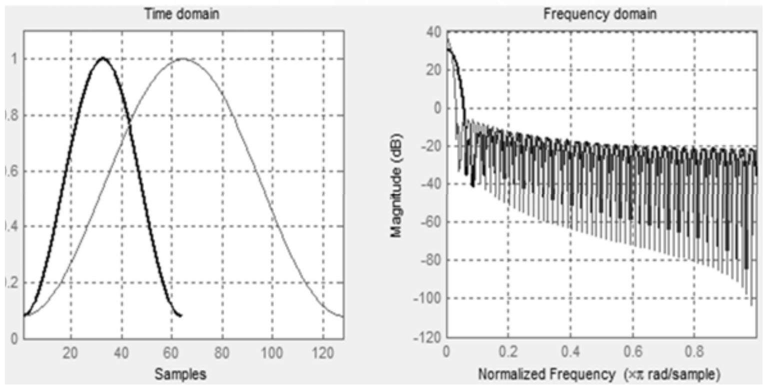
訊號長度與頻率解析度的關係



提升資料時間長度

- 資料時間長度越長 頻率解析度越高
- 增加資料點數或是 降低取樣頻率皆能 提高頻率解析度
- 以**Zero-padding**增加資料長度

訊號長度與頻率解析度的關係



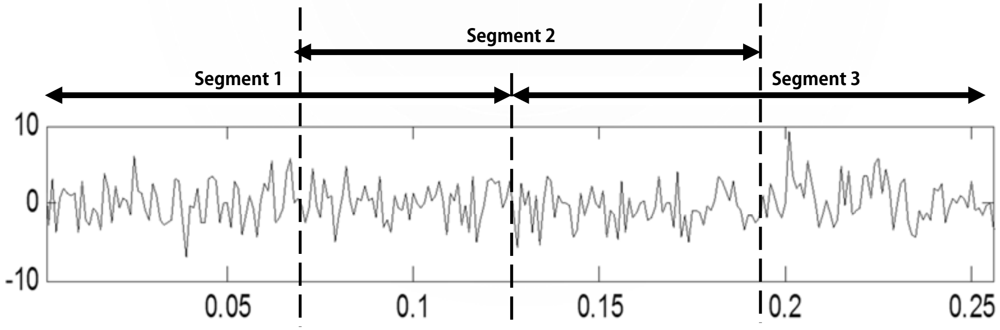
Hamming window (128 points
- **Mainlobe width (-3dB): 0.02**
- **Sidelobe attenuation: -42.6**

Hamming window (64 points)
- **Mainlobe width (-3dB): 0.03**
- **Sidelobe attenuation: -42.5**

在**command window** 下輸入**wintool**使用工具箱

透過WELCH METHOD進行頻譜分析

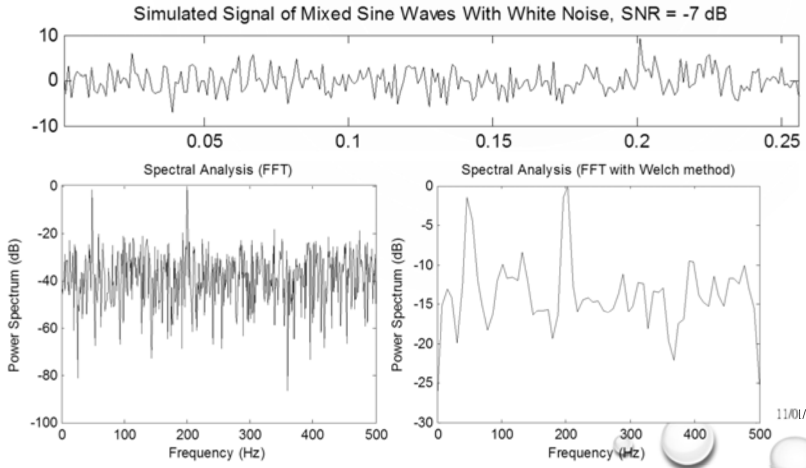
- 將訊號分段後計算各自的頻譜再平均



Overlapping的比例與Window的選擇都會影響到結果!

透過WELCH METHOD進行頻譜分析

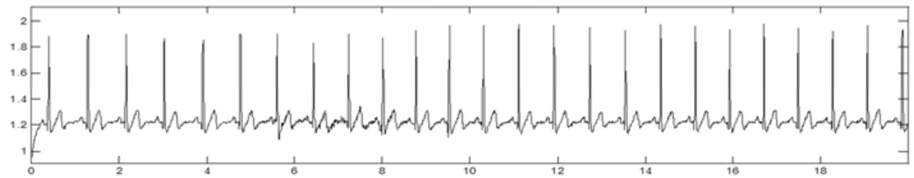
- 請開啟並執行**demodata_L6\Simulation_FFTspectrum.m**



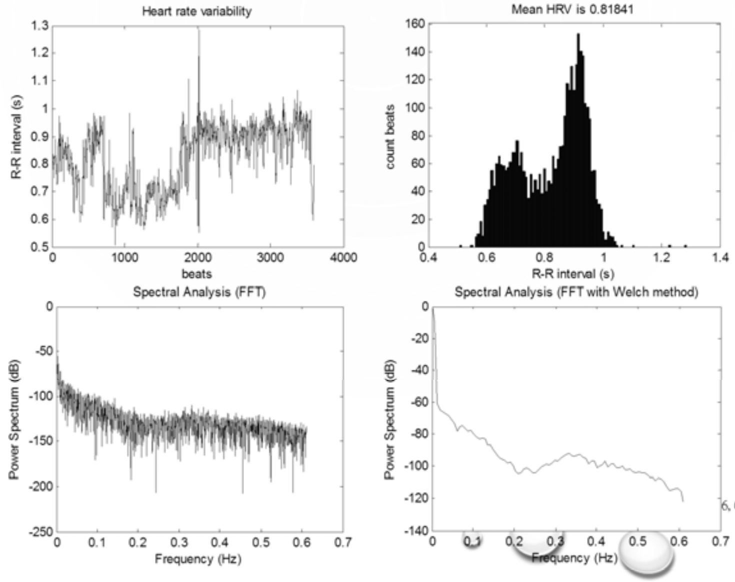
生理訊號頻譜分析實例

以心跳訊號ECG為例

- 請開啟並執行**demodata_L6\ECG\ECG_spectrum.m**
- 計算**Heart rate variability(R-R interval)**的頻譜分析

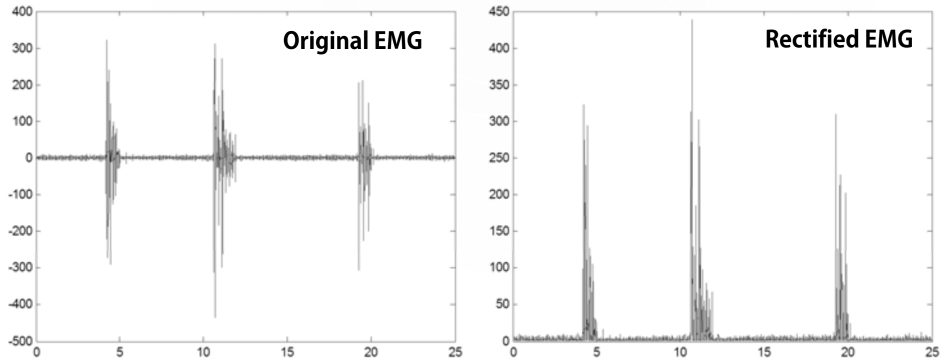


以心跳訊號ECG為例



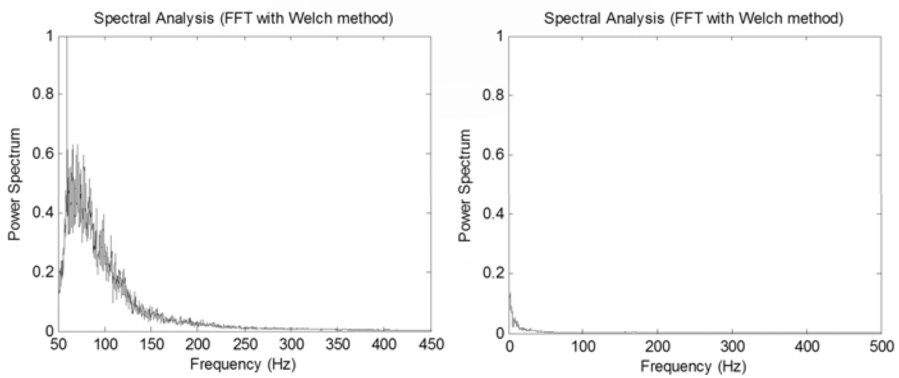
以肌電波訊號為例

- 請開啟並執行demodata_L6\EMG\EMG_spectrum.m



以肌電波訊號為例

- 請開啟並執行demodata_L6\EMG\EMG_spectrum.m



THE END

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