濾波器設計與應用

函示定義與傳呼

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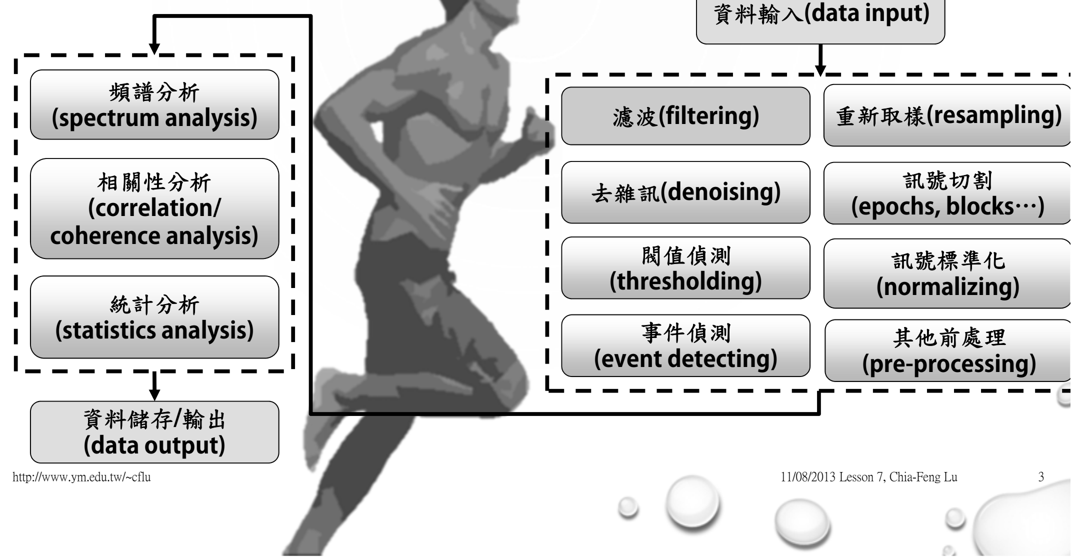
請先下載本週上課資料

- **http://www.ym.edu.tw/~cflu**
- 點選左欄〔課程資料〕
- 下載第**8**週上課資料〔**demodata_L7.zip**〕，檔案大小約**1MB**

訊號分析方法



本週課程內容

- 數位濾波器設計**(digital filter design)**
- 有限脈衝響應**(finite impulse response, FIR)**濾波器
- 無限脈衝響應**(infinite impulse response, IIR)**濾波器
- **MATLAB**函式定義與傳呼

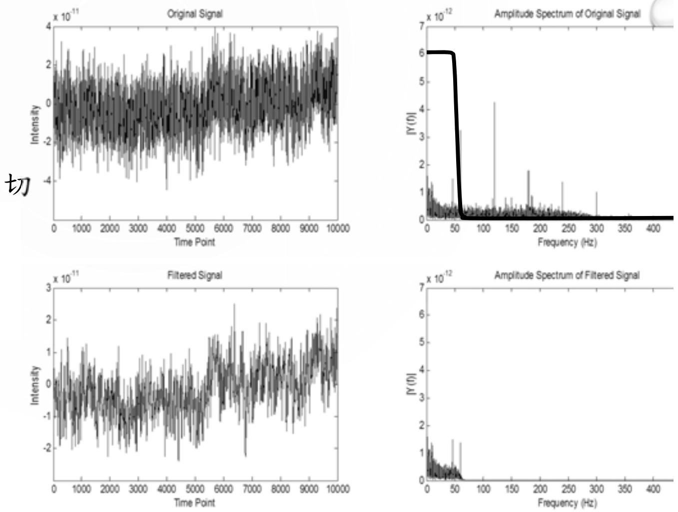
數位濾波器設計原理

濾波的目的

將訊號的頻譜成分作調整

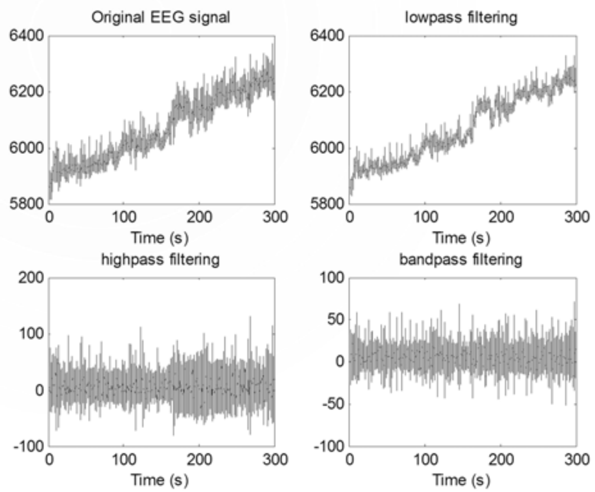
濾波器設計與頻譜分析關係密切

EEG signal



濾波的目的

- 鎖定特定頻帶觀察生理現象
- 移除高頻機器或環境雜訊
- 移除低頻訊號飄移成分



傅立葉轉換的特性

$$X(f)=\sum_{n=0}^{N-1}x(n)\cos(2\pi fn/N)-j\sum_{n=0}^{N-1}x(n)\sin(2\pi fn/N)$$
$$=\sum_{n=0}^{N-1}x(n)e^{-j2\pi fn/N}$$

	Time domain	Frequency domain
Convolution	$h(n)*x(n)$	$H(f)\cdot X(f)$
Time shifting	$x(n-k)$	$e^{-j2\pi fk/N}X(f)$

Z轉換(Z-TRANSFORM)的特性

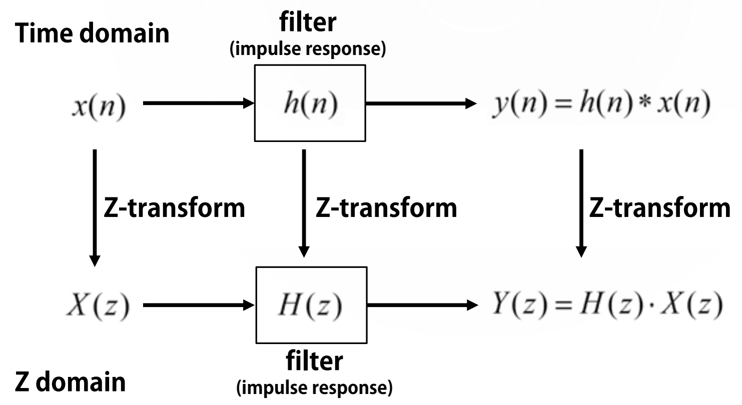
$$X(z) \equiv Z\{x(n)\} = \sum_{n=0}^{N-1} x(n) z^{-n}$$

z is an arbitrary complex value

如將z設為 $e^{j2\pi f/N}$ 即為傅立葉轉換!!

	Time domain	Z domain
Convolution	$h(n) * x(n)$	$H(z) \cdot X(z)$
Time shifting	$x(n-k)$	$z^{-k} X(z)$

濾波器的運算原理



轉換函數(DIGITAL TRANSFER FUNCTION)

$$H(z) = \frac{Y(z)}{X(z)} = \underbrace{\overbrace{\frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_K z^{-K}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_L z^{-L}}}^{\text{numerator}}}_{\text{denominator}} = \frac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}}, \text{usually } a_0 = 1$$

$$Y(z) = X(z) \cdot H(z) = X(z) \frac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}}$$

輸入輸出函式(INPUT-OUTPUT FUNCTION)

Z domain : $Y(z) = X(z) \frac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}}$

The order of filter
(the number of coefficients)

Time domain : $y(n) = \underbrace{\sum_{k=0}^{K} b_k x(n-k)}_{\text{numerator (moving average)}} - \underbrace{\sum_{l=1}^{L} a_l y(n-l)}_{\text{denominator (Autoregressive)}}$

輸入輸出函式(INPUT-OUTPUT FUNCTION)

ime domain $y(n)=\sum_{k=0}^{K} b_k x(n-k)-\sum_{l=1}^{L} a_l y(n-l)$

$$\sum_{l=0}^{L} a_l y(n-l)=\sum_{k=0}^{K} b_k x(n-k),\ \text{where } a_0=1$$

$$\sum_{l=0}^{L} a_l Z\{y(n-l)\}=\sum_{k=0}^{K} b_k Z\{x(n-k)\}$$ linearity of Z-transform

$$\sum_{l=0}^{L} a_l z^{-l} Z\{y(n)\}=\sum_{k=0}^{K} b_k z^{-k} Z\{x(n)\}$$ Time shifting of Z-transform

$$Z\{y(n)\}=Z\{x(n)\}\frac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}} \quad \text{or} \quad Y(z)=X(z)\frac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}}$$ # Z domain

轉換函數(濾波器)的頻譜分析

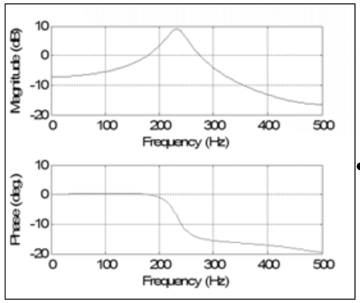
$$H(f)=\frac{Y(f)}{X(f)}=\frac{\sum_{k=0}^{K} b_k e^{-j2\pi fk/N}}{\sum_{l=0}^{L} a_l e^{-j2\pi fl/N}}=\frac{fft(b_k)}{fft(a_l)}$$

實例：

畫出下列轉換函數的頻譜與相位變化

$$H(z)=\frac{0.2+0.5z^{-1}}{1-0.2z^{-1}+0.8z^{-2}}$$

請執行demodata_L7\TransFun_spectrum.m



- 也可直接使用
  - [H,faxis] = freqz(b,a,N,'whole',samplerat

有限脈衝響應濾波器

FINITE IMPULSE RESPONSE, FIR FILTERS

有限脈衝濾波器FIR

- 轉換函數 $H(z)=\sum_{k=0}^{K} b_k z^{-k}$
- 輸入輸出函式

Z domain : $Y(z)=X(z)\sum_{k=0}^{K} b_k z^{-k}$

Time domain : $y(n)=\sum_{k=0}^{K} b_k x(n-k)$

numerator
(moving average)

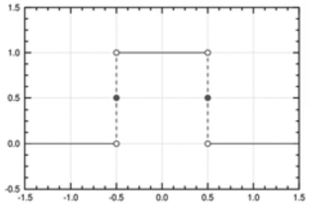
〔優點〕
- 穩定(stable)
- 線性相位變化(linear phase shift)

〔缺點〕
- 需要較高階數來滿足濾波條件
- 計算效率較低

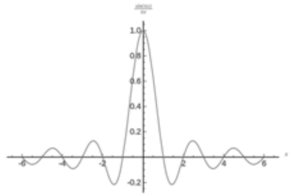
FIR濾波器設計

- 將設定好的頻率響應，反推算出轉換函數的係數**b**
- 係數的計算，可以透過對欲得到的頻率響應做反傳立葉轉換**(inverse Fourier transform)**取得

rectangular function
(Frequency domain)



sinc function
(Time domain)



$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

FIR濾波器設計

F_c is the cutoff frequency, T_s is the sample interval in second, L is the length of filter.

Lowpass

$$b(k) = \frac{\sin[2\pi f_c T_s (k - L/2)]}{\pi(k - L/2)}$$

Highpass

$$b(k) = \frac{\sin[\pi(k - L/2)]}{\pi(k - L/2)} - \frac{\sin[2\pi f_c T_s (k - L/2)]}{\pi(k - L/2)}$$

Bandpass

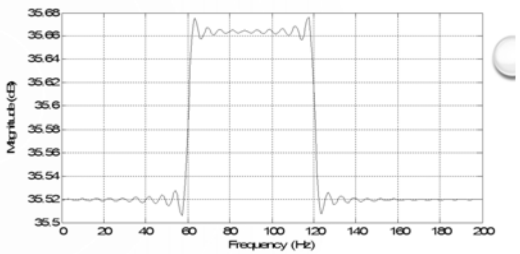
$$b(k) = \frac{\sin[2\pi f_H T_s (k - L/2)]}{\pi(k - L/2)} - \frac{\sin[2\pi f_L T_s (k - L/2)]}{\pi(k - L/2)}$$

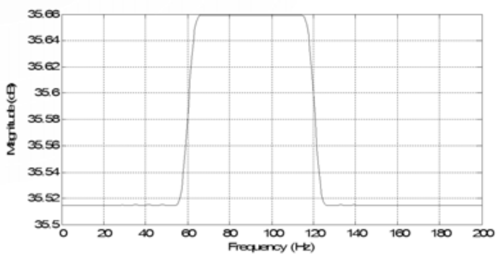
Bandstop

$$b(k) = \frac{\sin[2\pi f_L T_s (k - L/2)]}{\pi(k - L/2)} + \frac{\sin[\pi(k - L/2)]}{\pi(k - L/2)} - \frac{\sin[2\pi f_H T_s (k - L/2)]}{\pi(k - L/2)}$$

FIR濾波器設計

- **Window function**對於濾波器的助益
- 請執行**demodata_L7\FIR_Window.m**
- 比較下列兩者的差異：
  - **H = fft(b,N);**
  - **HW = fft(b.*hamming(L+1)',N);**





MATLAB濾波器設計函式

三階段設計流程

firpmord → **firpm** → **filtfilt, filter, conv**

1st-stage :
- 決定濾波器階數**(order)**與截斷頻率**(cutoff frequency)**

2nd-stage:
- 產生係數**b**序列

3rd-stage:
- 將濾波器套用到訊號上

二階段設計流程

fir1 → **filtfilt, filter, conv**

Data must have length more than 3 times filter order!!

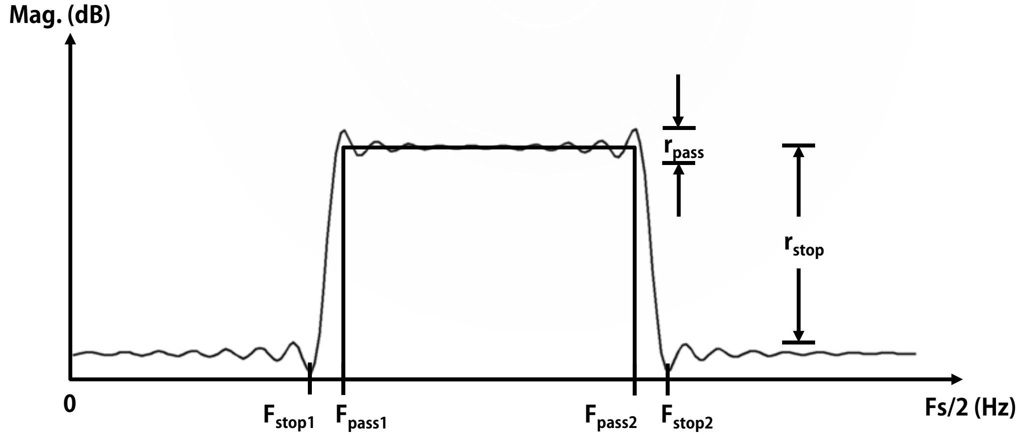
[MATLAB RULE]建立與傳呼函式(FUNCTION)

- 以mean.m為例
- 格式為:

```
function y = mean(x,dim)
%MEAN   Average or mean value.
%       For vectors, MEAN(X) is the mean value of the elements in X.
y = sum(x)/size(x,dim);
```

- 檔名需與function name相同
- 輸出變數一定要存在

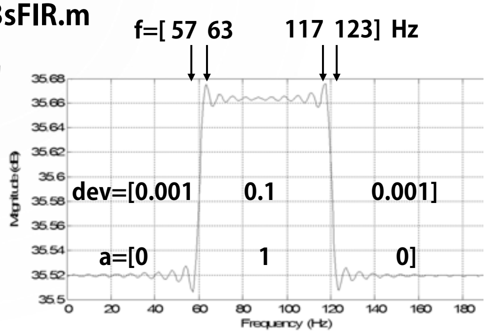
濾波器規格(FILTER SPECIFICATION)



MATLAB三階段FIR濾波處理

- 三階段FIR濾波處理，demodata_L7\filter_3sFIR.m

  [sigfilter,n] = filter_3sFIR(sig,f,a,dev,fs)

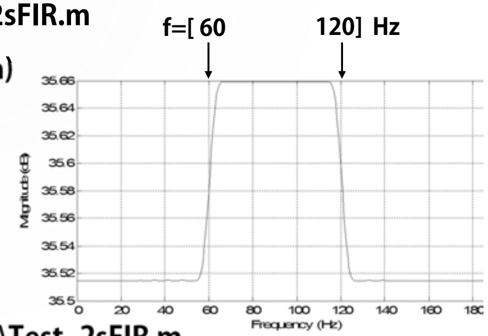


- 欲使用該函式，請開啟並執行demodata_L7\Test_3sFIR.m

MATLAB二階段FIR濾波處理

- 二階段FIR濾波處理，demodata_L7\filter_2sFIR.m

  sigfilter = filter_2sFIR(sig,f,fs,n,type,win)



- 欲使用該函式，請開啟並執行demodata_L7\Test_2sFIR.m

無限脈衝響應濾波器

INFINITE IMPULSE RESPONSE, IIR FILTERS

無限脈衝濾波器IIR

- 轉換函數 $H(z)=\dfrac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}}$
- 輸入輸出函式

Z domain : $Y(z)=X(z)\dfrac{\sum_{k=0}^{K} b_k z^{-k}}{\sum_{l=0}^{L} a_l z^{-l}}$

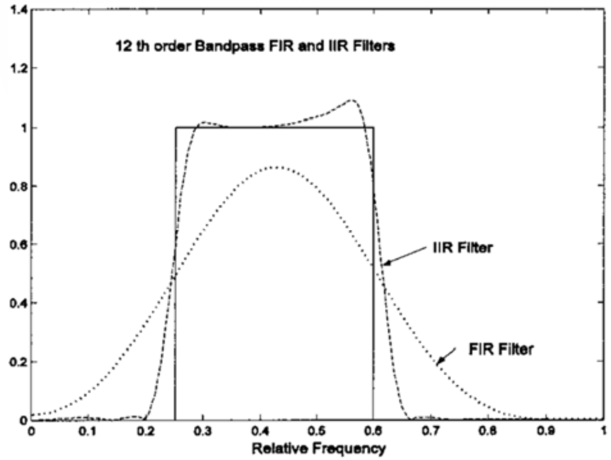
Time domain : $y(n)=\underbrace{\sum_{k=0}^{K} b_k x(n-k)}_{\text{numerator}}-\underbrace{\sum_{l=1}^{L} a_l y(n-l)}_{\text{denominator}}$

numerator (moving average) **denominator** (Autoregressive)

〔優點〕
- 僅需較低階數來滿足濾波條件
- 計算效率較高，但運算要持續用到整段訊號

〔缺點〕
- 可能不穩定(unstable)，分母項為0
- 非線性相位變化(nonlinear phase shift

同階數FIR與IIR比較



MATLAB濾波器設計函式

三階段設計流程

buttord, cheb1ord, cheb2ord, ellipord → butter, cheb1, cheb2, ellip → filtfilt

1st-stage :
- 決定濾波器階數(order)與截斷頻率(cutoff frequency)

2nd-stage:
- 產生係數b與a序列

3rd-stage:
- 將濾波器套用到訊號上

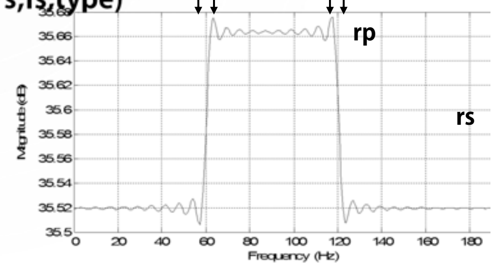
二階段設計流程

butter, cheb1, cheb2, ellip → filtfilt

Data must have length more than 3 times filter order!!

MATLAB三階段IIR濾波處理

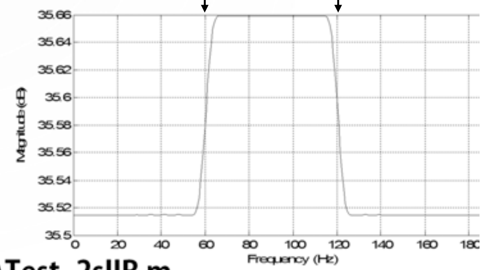
- 三階段IIR濾波處理，**demodata_L7\filter_3sIIR.m**
  [sigfilter, n]=filter_3sIIR(sig,wp,ws,rp,rs,fs,type)



- 欲使用該函式，請開啟並執行**demodata_L7\Test_3sIIR.m**

MATLAB二階段IIR濾波處理

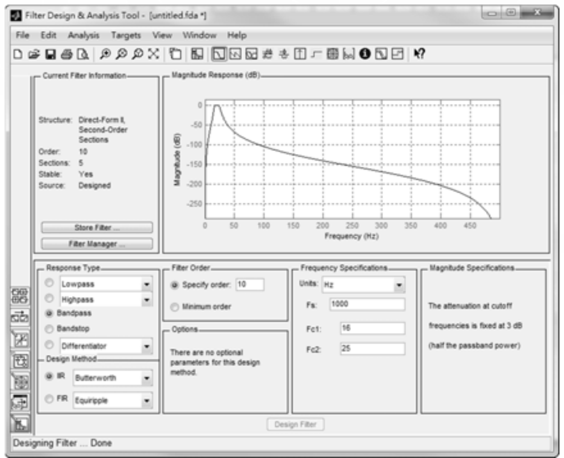
- 二階段IIR濾波處理，**demodata_L7\filter_2sIIR.m**
  sigfilter = filter_2sIIR(sig,f,fs,n,type)



- 欲使用該函式，請開啟並執行**demodata_L7\Test_2sIIR.m**

MATLAB濾波器設計工具箱

- 在**command window**下輸入**fdatool**呼叫濾波器設計工具箱



THE END

http://www.ym.edu.tw/~cflu