時頻分析

短時距傅立葉轉換、小波轉換

盧家鋒 助理教授

國立陽明大學 物理治療暨輔助科技學系

alvin4016@ym.edu.tw

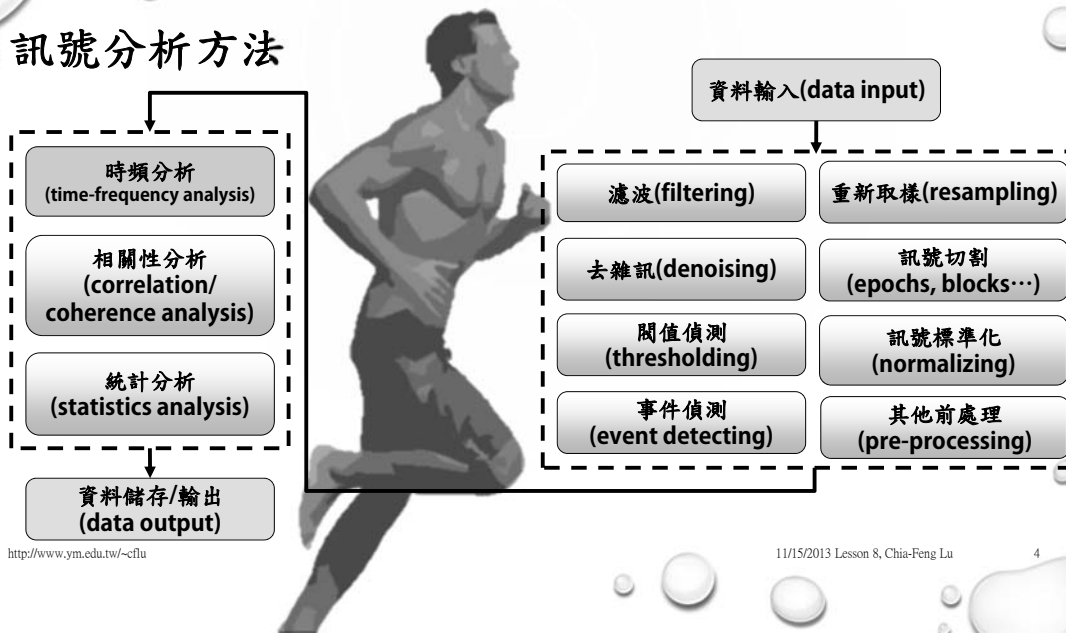
請先下載本週上課資料

- http://www.ym.edu.tw/~cflu
- 點選左欄〔課程資料〕
- 下載第9週上課資料〔demodata_L8.zip〕，檔案大小約100MB

本週課程內容

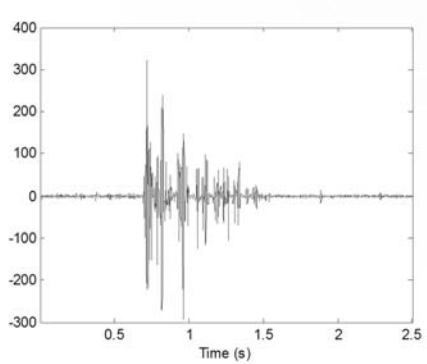
- 短時距傅立葉轉換(Short time Fourier transform)
- 小波轉換(Wavelet transform)
- 生理訊號時頻分析實例

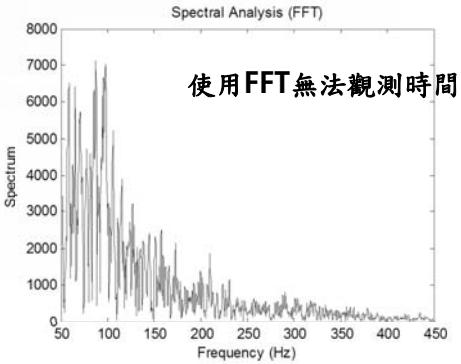
訊號分析方法



時頻分析(TIME-FREQUENCY ANALYSIS)

- 針對**non-stationary**訊號(大多的生理訊號皆是如此)，頻率成分會隨時間做變動

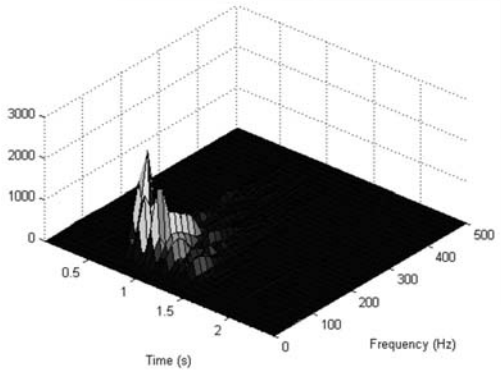




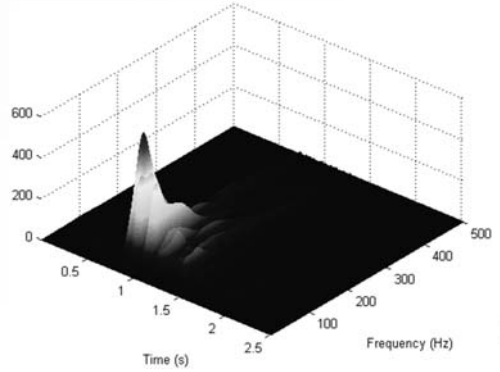
使用FFT無法觀測時間的訊息量!

時頻分析

Spectrogram (STFT)

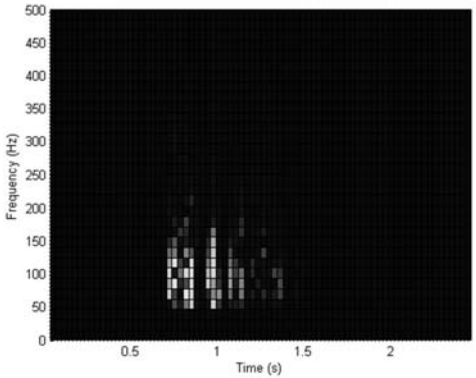


Continuous wavelet transform

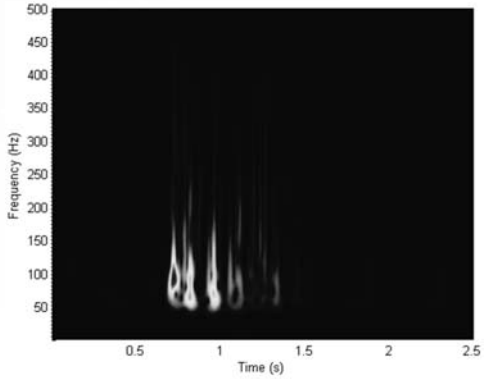


時頻分析

Spectrogram (STFT)

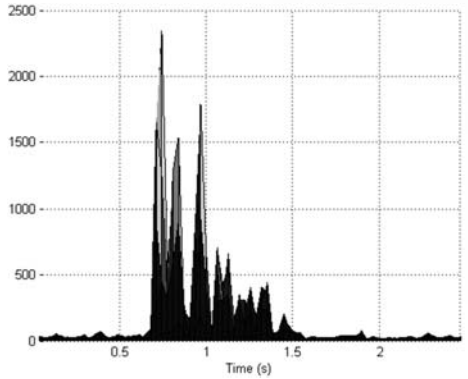


Continuous wavelet transform

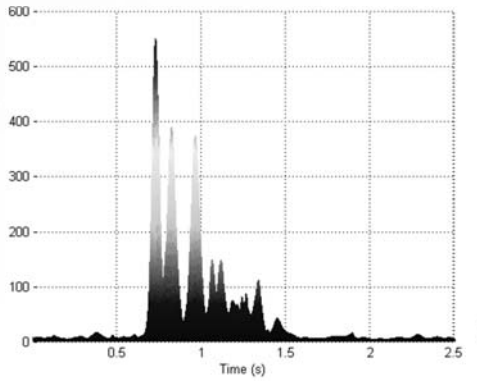


時頻分析

Spectrogram (STFT)

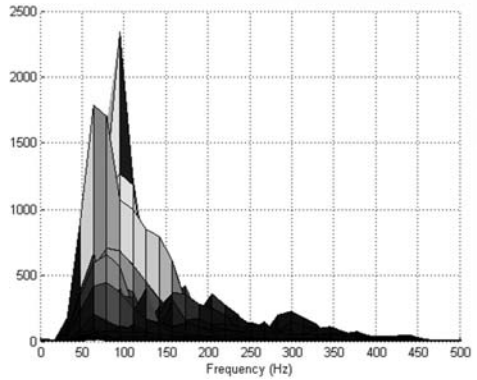


Continuous wavelet transform

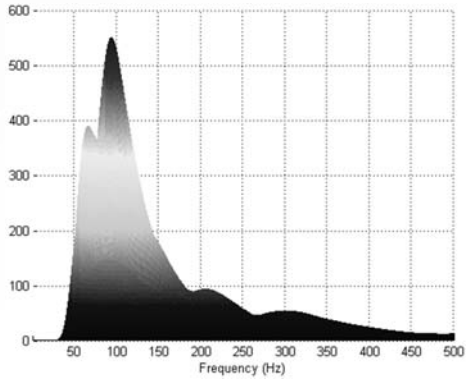


時頻分析

Spectrogram (STFT)



Continuous wavelet transform



短時距傅立葉轉換

SHORT TIME FOURIER TRANSFORM

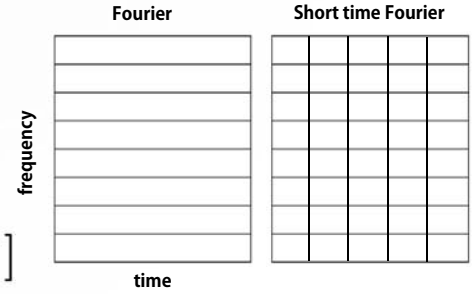
短時距傅立葉轉換 (SHORT TIME DISCRETE FOURIER TRANSFORM)

- 傅立葉轉換

$$S(f)=\sum_{n=0}^{N-1}s(n)e^{-j2\pi fn/N}$$

- 套用**window function**於傅立葉轉換上

$$S(f,k)=\sum_{n=0}^{N-1}s(n)\left[W(n-k)e^{-j2\pi fn/N}\right]$$



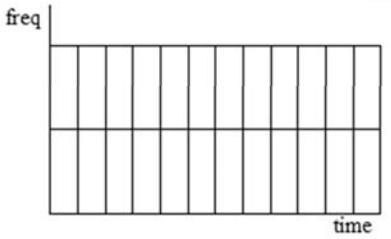
利用**STFT**的時頻分析方式又稱為**Spectrogram**

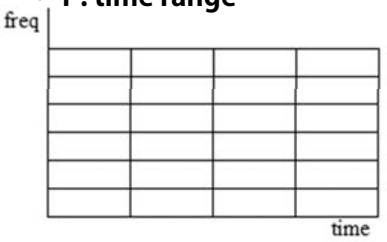
時間頻率解析度的交互關係

- **Time-frequency trade off**

$$\text{uncertainty principle}: BT \geq \frac{1}{4\pi}$$

- **B : frequency bandwidth**
- **T : time range**





Window越長（資料點數越多），頻率解析度越高

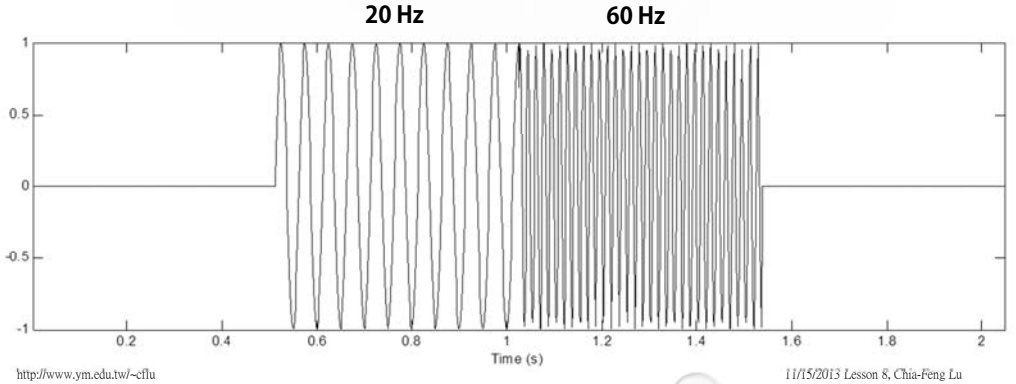
[MATLAB RULE]使用SPECTROGRAM

- help spectrogram
  - [S, F, T] = SPECTROGRAM(X,WINDOW,NOVERLAP,NFFT,Fs);

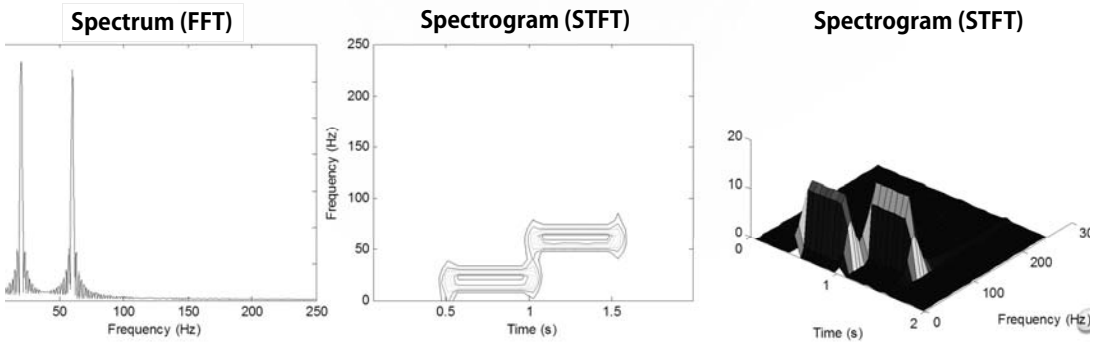
- 和welch method比較看看有何不同?
  - [Pxx,F] = PWELCH(X,WINDOW,NOVERLAP, NFFT,Fs);

使用STFT分析STEP CHANGE的SINE訊號

- 請開啟並執行demodata_L8\STFT_stepchange.m

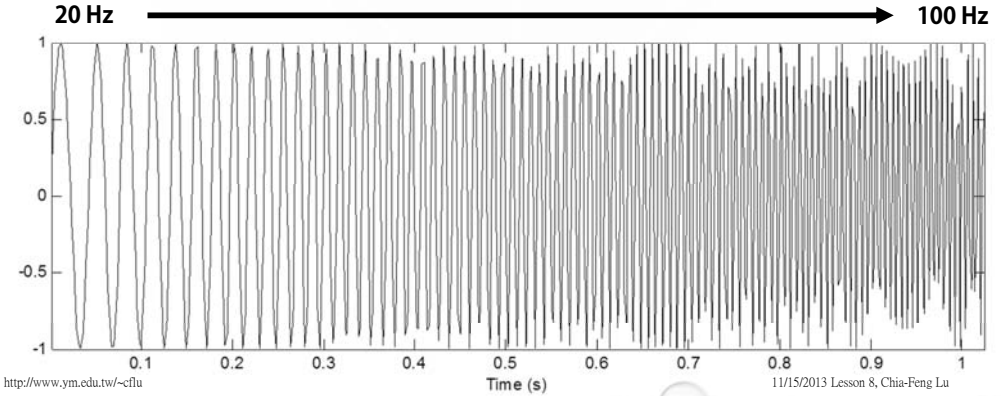


觀察STFT得到結果在時間與頻率域的特性

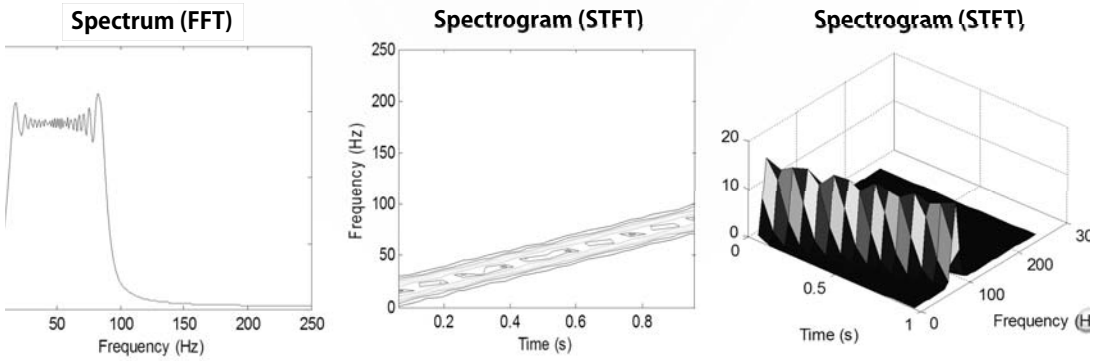


使用STFT分析CHIRP SIGNAL

- 請開啟並執行demodata_L8\STFT_chirpsignal.m



觀察STFT得到結果在時間與頻率域的特性

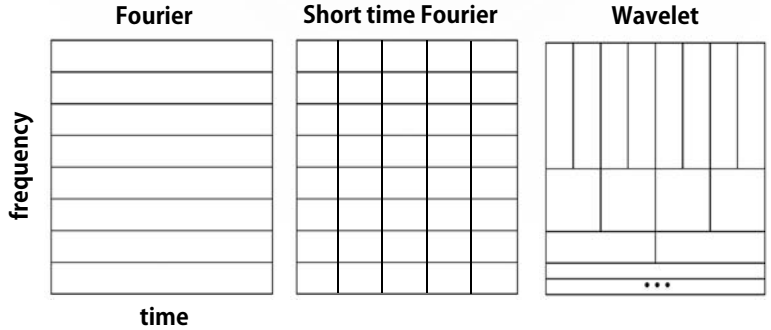


小波轉換

CONTINUOUS WAVELET TRANSFORM

STFT足夠了嗎?

- 高頻的成分僅需較段的時間長度來呈現（訊號變動快）
- 低頻的成分需要較長的時間長度來量測（訊號變動慢）



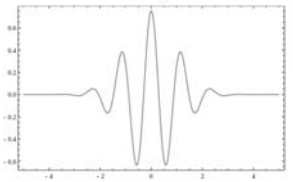
連續小波轉換(CONTINUOUS WAVELET TRANSFORM)

- 用小波基底**(wavelet)**來重現訊號

$$W(a,b)=\int_{-\infty}^{\infty}x(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)dt,$$

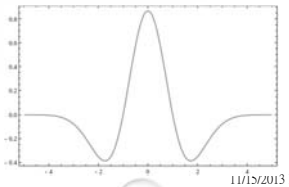
- **Morlet Wavelet**

$$\psi(x)=e^{-x^2}\cos(\pi\sqrt{\frac{2}{\ln 2}}x)$$



- **Mexican hat wavelet**

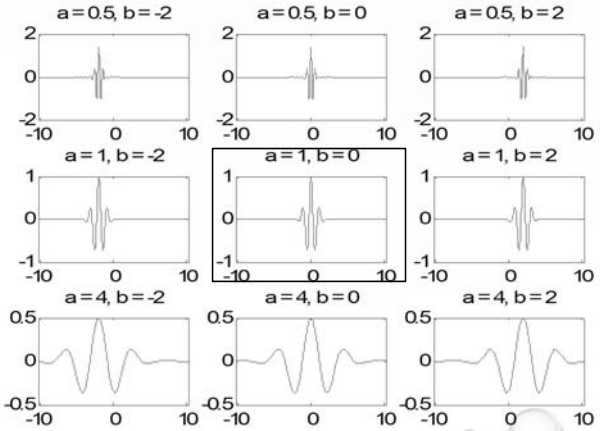
$$\psi(x)=(1-2x^2)e^{-x^2}$$



小波的特性

- 延展(dilation，a)與平移(translation，b)

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$



Morlet wavelet

$\psi_{1,0}$: mother wavelet

小波的時間與頻率特性

- **Center time**

$$t_0=\frac{\int_{-\infty}^{\infty}t|\psi(t)|^2dt}{\int_{-\infty}^{\infty}|\psi(t)|^2dt}$$

time range

$$\Delta t_\psi=\sqrt{\frac{\int_{-\infty}^{\infty}(t-t_0)^2|\psi(t)|^2dt}{\int_{-\infty}^{\infty}|\psi(t)|^2dt}}$$

- **Center frequency**

$$\omega_0=\frac{\int_{-\infty}^{\infty}\omega|\Psi(\omega)|^2d\omega}{\int_{-\infty}^{\infty}|\Psi(\omega)|^2d\omega}$$

frequency range (bandwidth)

$$\Delta\omega_\psi=\sqrt{\frac{\int_{-\infty}^{\infty}(\omega-\omega_0)^2|\Psi(\omega)|^2d\omega}{\int_{-\infty}^{\infty}|\Psi(\omega)|^2d\omega}}$$

時間頻率解析度的交互關係

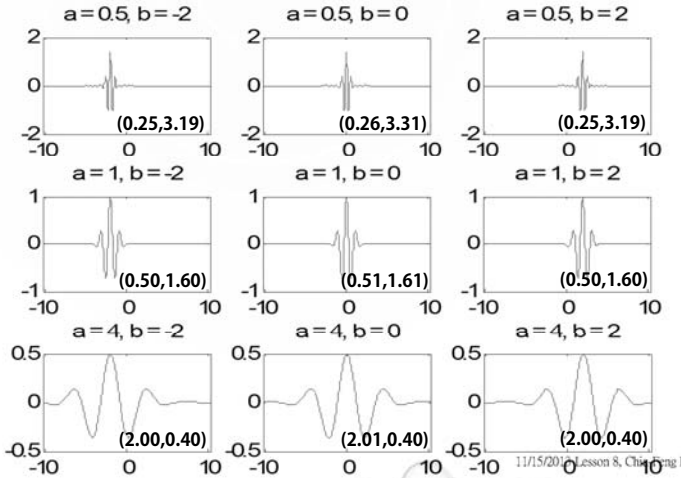
Morlet wavelet

$(\Delta t_\psi(a),\Delta\omega_\psi(a))$

$a\uparrow,\Delta\omega_\psi(a)\downarrow$

$\Delta\omega_\psi(a)=\Delta\omega_\psi/|a|$

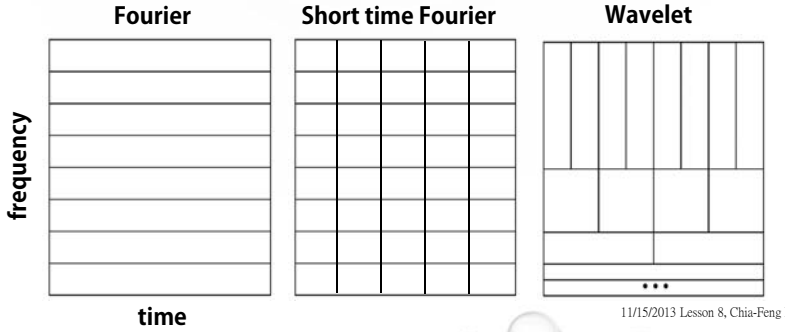
$\Delta\omega_\psi(a)\cdot\Delta\omega_\psi(a)\approx 0.80$



時間頻率解析度的交互關係

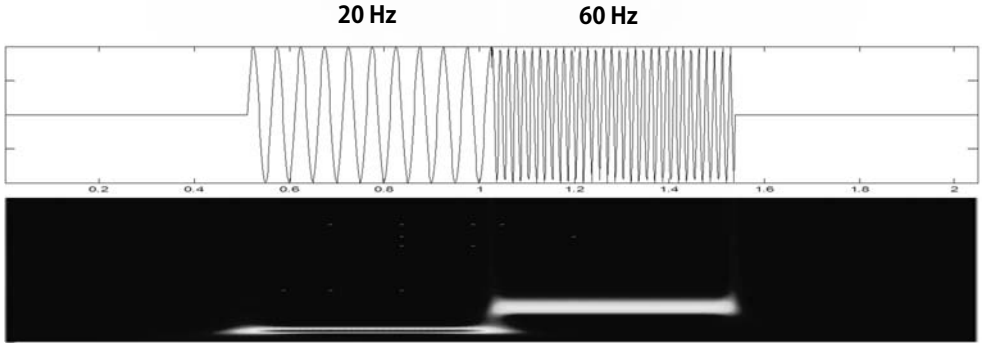
- **Time-frequency trade off**

$$\Delta\omega_\psi(a)\Delta t_\psi(a)=\Delta\omega_\psi\Delta t_\psi=\text{constant}_\psi$$

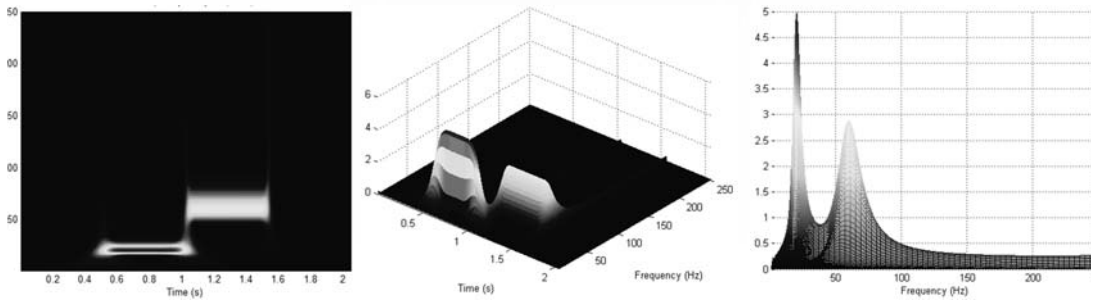


使用CWT分析STEP CHANGE的SINE訊號

- 請開啟並執行demodata_L8\MWT_stepchange.m

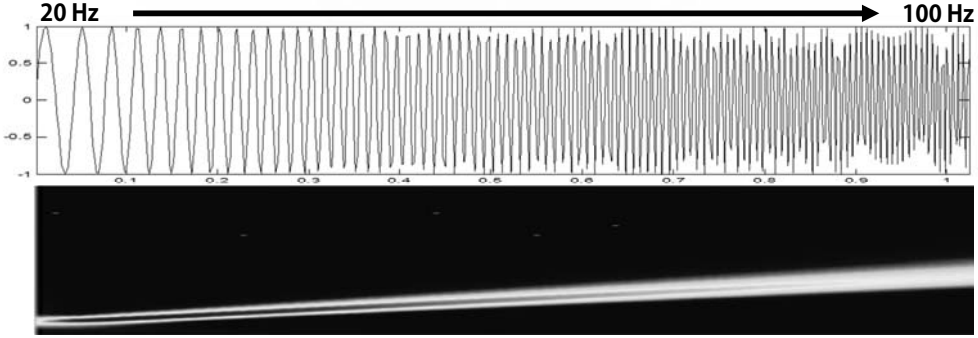


觀察CWT得到結果在時間與頻率域的特性

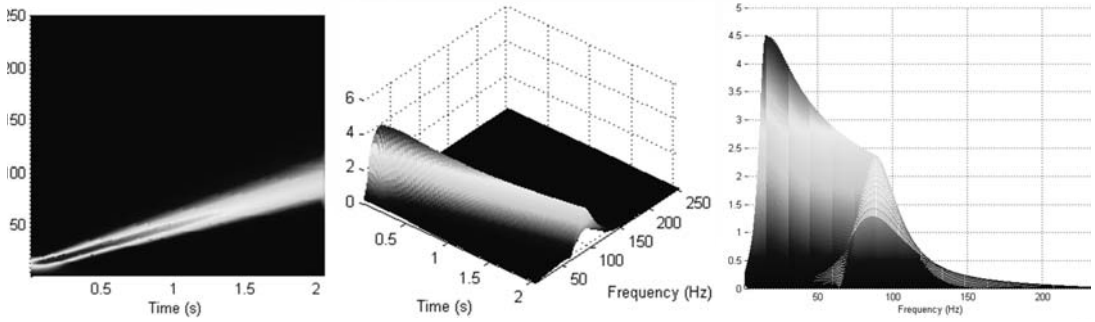


使用CWT分析CHIRP SIGNAL

- 請開啟並執行demodata_L8\MWT_chirpsignal.m



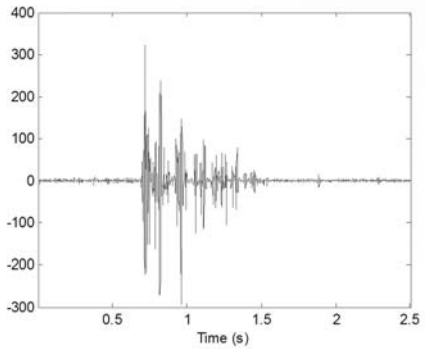
觀察STFT得到結果在時間與頻率域的特性

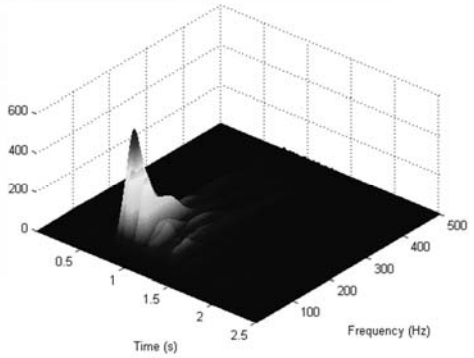


生理訊號頻譜分析實例

以EMG肌電訊號為例

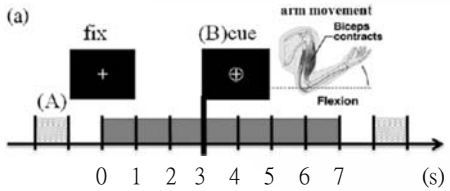
- 請開啟並執行demodata_L8\EMG\EMG_spectrum.m

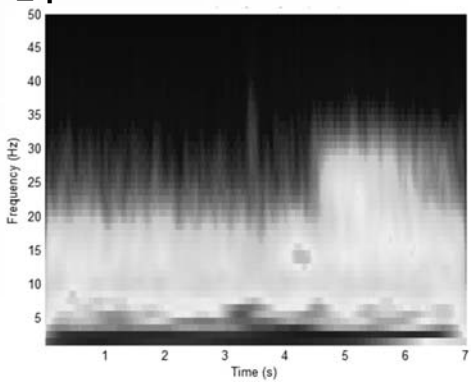




以EEG腦電波訊號為例

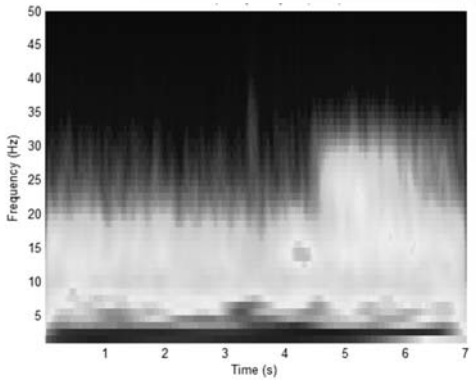
- 請開啟並執行demodata_L8\EEG\EEG_spectrum.m



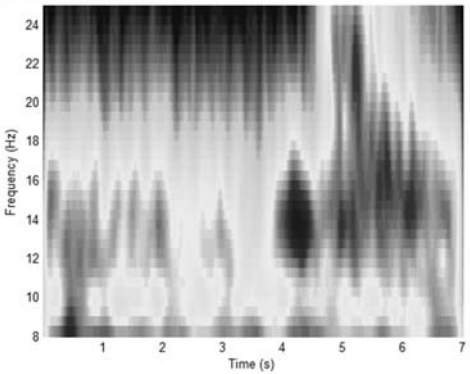


以EEG腦電波訊號為例

1:1:50 Hz



8:0.5:25 Hz



THE END

http://www.ym.edu.tw/~cflu