訊號同調性分析

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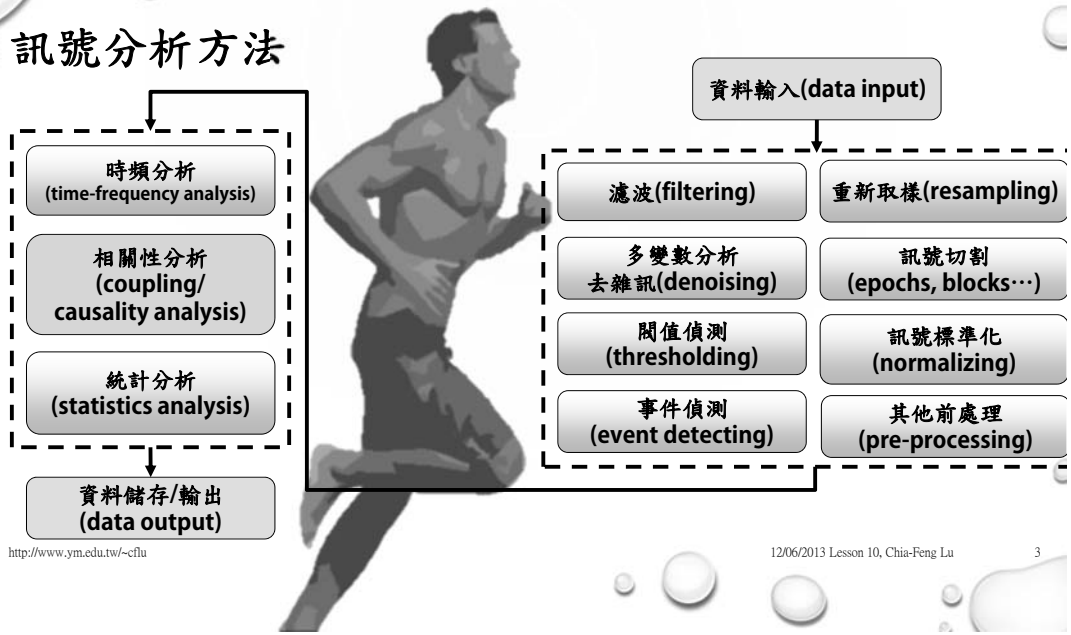
請先下載本週上課資料

- **http://www.ym.edu.tw/~cflu**
- 點選左欄〔課程資料〕
- 下載第**12**週上課資料〔**demodata_L10.zip**〕，檔案大小約**100MB**

訊號分析方法



本週課程內容

相關性**(Coupling)**

〔時間域〕

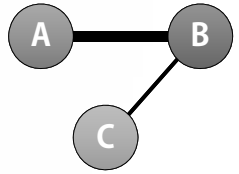
- 相關係數**(Correlation coefficient)**
- 共變異數**(Covariance)**

〔頻率域〕

- 同調性**(Coherence)**
- 部分同調性**(Partial coherence)**

〔時間與頻率成分〕

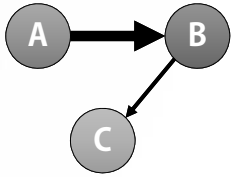
- 時頻共訊息法**(Time-frequency cross mutual information)**



相關課程內容

因果關係(Causality)

- 定向同調性(Directed coherence)
- 定向轉換函數(Directed transfer function)
- 部分定向同調性(Partial directed coherence)
- Granger因果關係(Granger causality)
- 動態因果模型(Dynamic causal modeling)



同調性分析

FFT-BASED WELCH METHOD

相關性與同調性

- Pearson相關性係數

$$r_{xy} = \frac{\sum_{t=1}^{N}(x(t)-\bar{x})(y(t)-\bar{y})}{\sqrt{\sum_{t=1}^{N}(x(t)-\bar{x})^2 \sum_{t=1}^{N}(y(t)-\bar{y})^2}}$$

- 同調性(Coherence)

magnitude-squared coherence

$$coh_{xy}(f) = \left| \frac{S_{xy}(f)}{\sqrt{S_{xx}(f)S_{yy}(f)}} \right|^2$$

$S_{xx}(f)$ is the power spectrum density of x
$S_{yy}(f)$ is the power spectrum density of y
$S_{xy}(f)$ is the cross-spectral density between x and y

同調性是頻率的函數，數值介於0~1之間，評估在特定頻帶上X與Y的頻率線性相關性。

相關性與同調性

- Pearson相關性係數

$$r_{xy} = \frac{\sum_{t=1}^{N}(x(t)-\bar{x})(y(t)-\bar{y})}{\sqrt{\sum_{t=1}^{N}(x(t)-\bar{x})^2 \sum_{t=1}^{N}(y(t)-\bar{y})^2}}$$

- 同調性(Coherence)

magnitude-squared coherence

$$coh_{xy}(f) = \left| \frac{\sum_{i=1}^{N}(X_i-\bar{X})(Y_i-\bar{Y})^*}{\sqrt{\sum_{i=1}^{N}(X_i-\bar{X})(X_i-\bar{X})^* \sum_{i=1}^{N}(Y_i-\bar{Y})(Y_i-\bar{Y})^*}} \right|^2$$

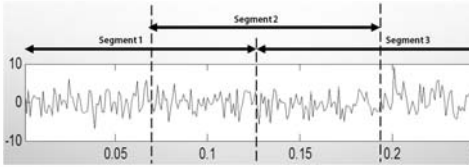
X_i and Y_i are complex values of spectrum recorded on epo
Total number of epochs is N
$\bar{X}$ and $\bar{Y}$ are the distribution means

如何計算同調性

- 方法一：
  - 透過傅立葉轉換得到$X(f)$與$Y(f)$
  - 計算S_{xx}, S_{yy}, S_{xy}
- help mscohere

- 方法二：
  - 計算X與Y之多變數自迴歸模型係數 (multivariate autoregressive model)
  - 透過係數計算S_{xx}, S_{yy}, S_{xy}
- 使用eMVAR toolbox
  - http://www.science.unitn.it/~nollo/research/sigpro/eMVAR.html

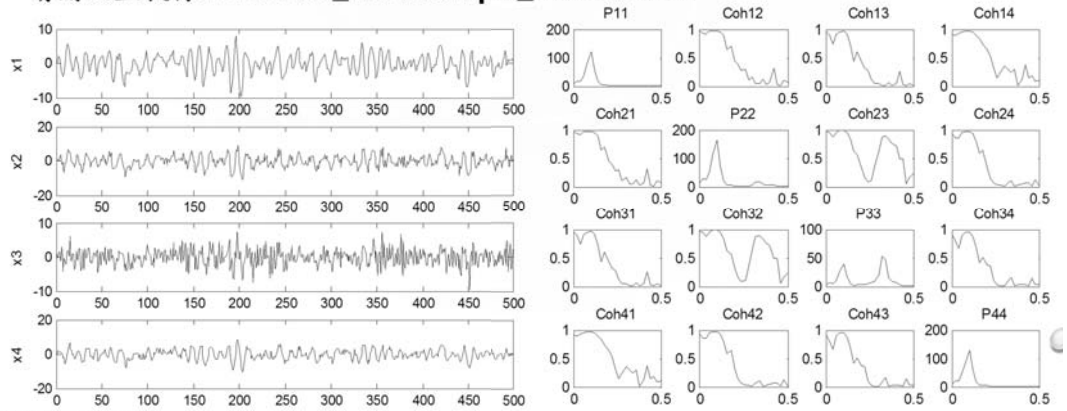
[MATLAB RULE]使用MSCOHERE



- help mscohere
  - [Sxy,F] = MSCOHERE(X,Y)
  - [Sxy,F] = MSCOHERE(X,Y,WINDOW,NOVERLAP,NFFT,Fs)
    - Estimate the magnitude-squared coherence of the system with input X and output Y using Welch's averaged periodogram method.
    - The inputs are the same with that of pwelch.

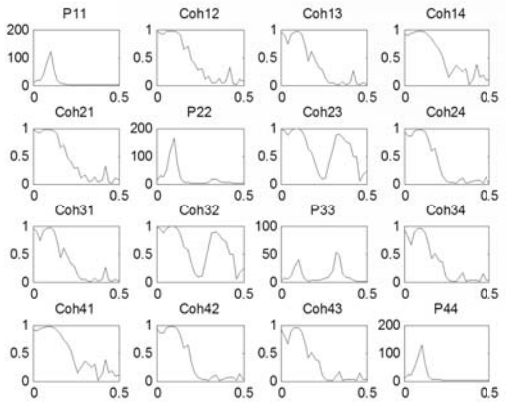
使用MSCOHERE計算同調性

- 請開啟並執行demodata_L10\example_mscohere.m

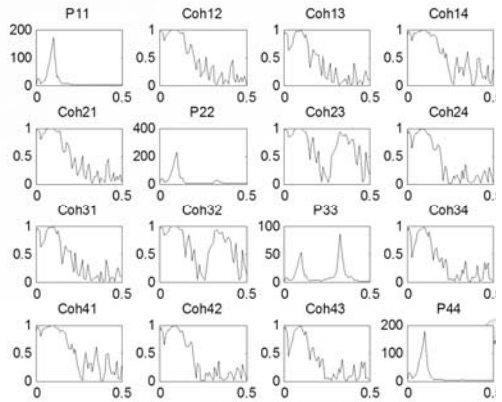


調控SEGMENT參數觀察得到的同調性變化

noverlap = 25, nfft = 50 → 19 segments

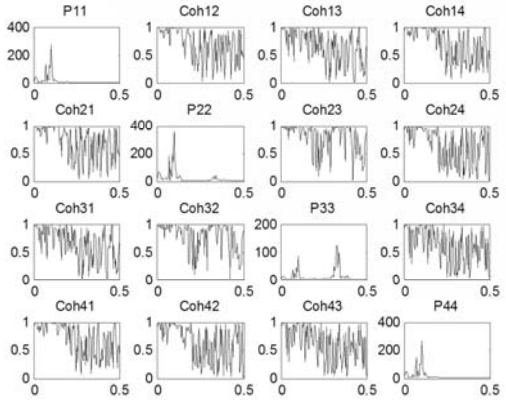


noverlap = 50, nfft = 100 → 9 segments

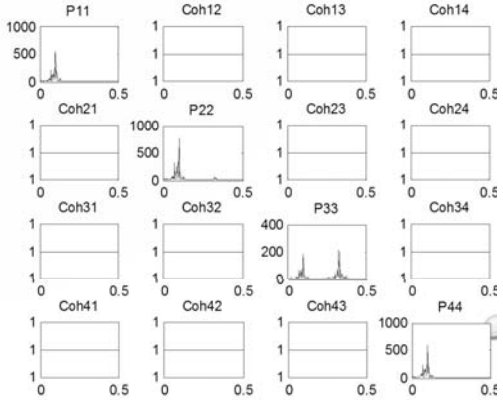


調控SEGMENT參數觀察得到的同調性變化

noverlap = 0, nfft = 250 → 2 segments

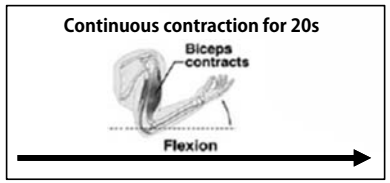


noverlap = 0, nfft = 500 → 1 segment

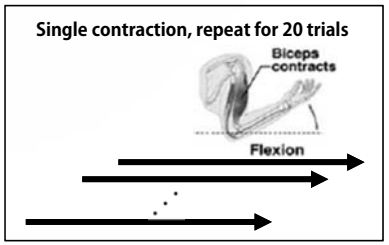


調控SEGMENT參數觀察得到的同調性變化

- 以連續序列訊號來看，訊號必需是**stationary!!**
  - 對時間作加總



- 以事件相關訊號來看，訊號必需在不同**epochs**間維持足夠相似性**!!**
  - 對**epoch**作加總，**event-related coherence**



同調性的統計顯著存在

- **Theoretical threshold, Rosenberg et al. 1989**

$$1-\left(1-\frac{\alpha}{100}\right)^{\frac{1}{(n-1)}}$$ **The confidence limit at α % based on *n* observations/segments/epochs**

- 例如：在有**n=19**個量測量，欲達到**95%**信心水準所需的同調性閥值為

$$1-\left(1-\frac{95}{100}\right)^{\frac{1}{(19-1)}}=0.1533$$

同調性分析

MULTIVARIATE AUTOREGRESSIVE (MVAR) MODEL

多變數自迴歸模型

- **Multivariate Autoregressive (MVAR) model**

$$\mathbf{X}(n)=\sum_{k=1}^{p}\mathbf{A}(k)\mathbf{X}(n-k)+\mathbf{U}(n)$$

  - ***p*是模型階數(model order)，定義最多前幾個資料點(maximum lag)會被用於考慮交互作用。**
  - **A是模型係數(model coefficients)，為k的函數**
  - **U為互不相關的高斯雜訊(uncorrelated while noise)，又稱為innovation process**

多變數自迴歸模型實例

以***p*=2為例，** $\mathbf{X}(n)=\sum_{k=1}^{2}\mathbf{A}(k)\mathbf{X}(n-k)+\mathbf{U}(n)$

$$x_1(n)=2\rho_1\cos(2\pi f_1)x_1(n-1)-\rho_1^2x_1(n-2)+u_1(n)$$
$$x_2(n)=x_1(n-1)+2\rho_2\cos(2\pi f_2)x_2(n-1)+0.5x_3(n-1)-\rho_2^2x_2(n-2)+u_2(n)$$
$$x_3(n)=0.5x_2(n-1)+2\rho_3\cos(2\pi f_3)x_3(n-1)+0.5x_2(n-2)-\rho_3^2x_3(n-2)+u_3(n)$$
$$x_4(n)=2\rho_4\cos(2\pi f_4)x_4(n-1)+x_1(n-2)-\rho_4^2x_4(n-2)+u_4(n)$$

多變數自迴歸模型實例

以***p*=2為例，** $\mathbf{X}(n)=\sum_{k=1}^{2}\mathbf{A}(k)\mathbf{X}(n-k)+\mathbf{U}(n)$

$$\mathbf{A}(1)=\begin{bmatrix}2\rho_1\cos(2\pi f_1) & 0 & 0 & 0\\ 1 & 2\rho_2\cos(2\pi f_2) & 0.5 & 0\\ 0 & 0.5 & 2\rho_3\cos(2\pi f_3) & 0\\ 0 & 0 & 0 & 2\rho_4\cos(2\pi f_4)\end{bmatrix},\quad\begin{bmatrix}x_1(n-1)\\ x_2(n-1)\\ x_3(n-1)\\ x_4(n-1)\end{bmatrix}$$

$$\mathbf{A}(2)=\begin{bmatrix}-\rho_1^2 & 0 & 0 & 0\\ 0 & -\rho_2^2 & 0 & 0\\ 0 & 0.5 & -\rho_3^2 & 0\\ 1 & 0 & 0 & -\rho_4^2\end{bmatrix},\quad\begin{bmatrix}x_1(n-2)\\ x_2(n-2)\\ x_3(n-2)\\ x_4(n-2)\end{bmatrix}$$

$a_{21}(1)$, $a_{32}(1)$, $a_{23}(1)$, $a_{41}(2)$, $a_{32}(2)$
$x_1\rightarrow x_2$, $x_2\rightarrow x_3$, $x_3\rightarrow x_2$, $x_1\rightarrow x_4$

- 請開啟並執行**demodata_L10\create_SimSignal.m**

利用迴歸係數估算頻譜

$$\mathbf{S}(f)=\mathbf{H}(f)\mathbf{\Sigma}\mathbf{H}^H(f),\ \mathbf{P}(f)=\overline{\mathbf{A}}^H(f)\mathbf{\Sigma}^{-1}\overline{\mathbf{A}}(f)$$

$$\mathbf{H}(f)=\overline{\mathbf{A}}(f)^{-1}=[\mathbf{I}-\mathbf{A}(f)]^{-1}$$

H is the transfer matrix,
the superscript H is the Hermitian transpose

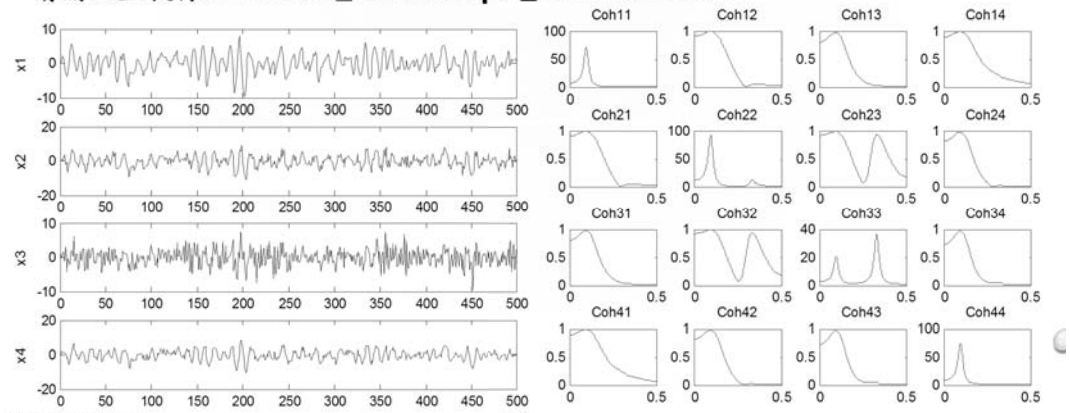
$$\mathbf{A}(f)=\sum_{k=1}^{p}\mathbf{A}(k)e^{-j2\pi fkt}$$

[MATLAB RULE]使用EMVAR TOOLBOX

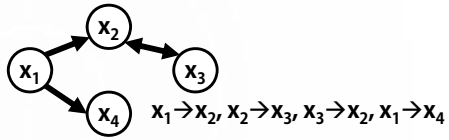
- [pottaic,pottmdl,aic,mdl] = mos_idMVAR(X,pmax,idMode)
  - Model order selection/optimization
- [Am,S,Yp,Up]=idMVAR(X,p,Mode)
  - Identification of MVAR coefficients and covariance
- [DC,DTF,PDC,GPDC,COH,PCOH,PCOH2,H,S,P,f] = fdMVAR(Am,Su,N,Fs)
  - Frequency domain MVAR analysis

使用MVAR計算同調性

- 請開啟並執行demodata_L10\example_MVARcohere.m

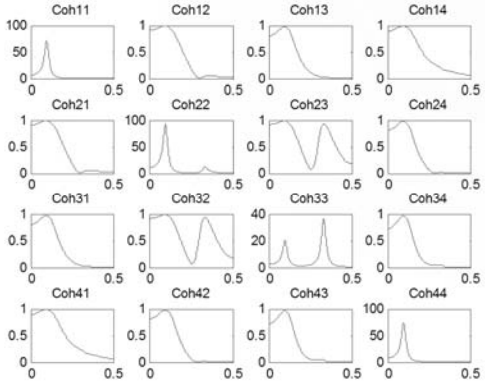


比較COH與PCOH結果

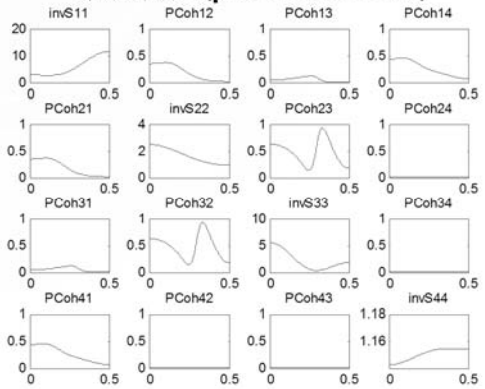


$x_1 \rightarrow x_2$, $x_2 \rightarrow x_3$, $x_3 \rightarrow x_2$, $x_1 \rightarrow x_4$

同調性(coherence)



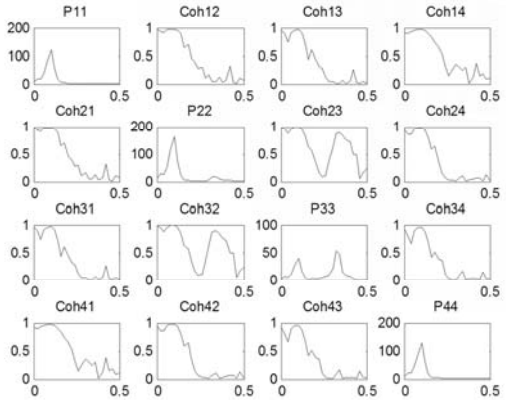
部分同調性(partial coherence)



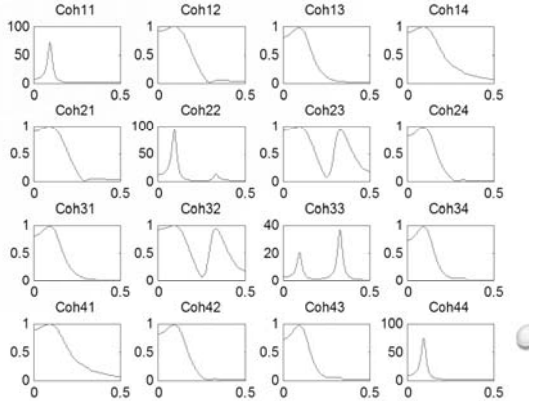
比較FFT與MVAR同調性結果

Finer frequency resolution!

mscohere



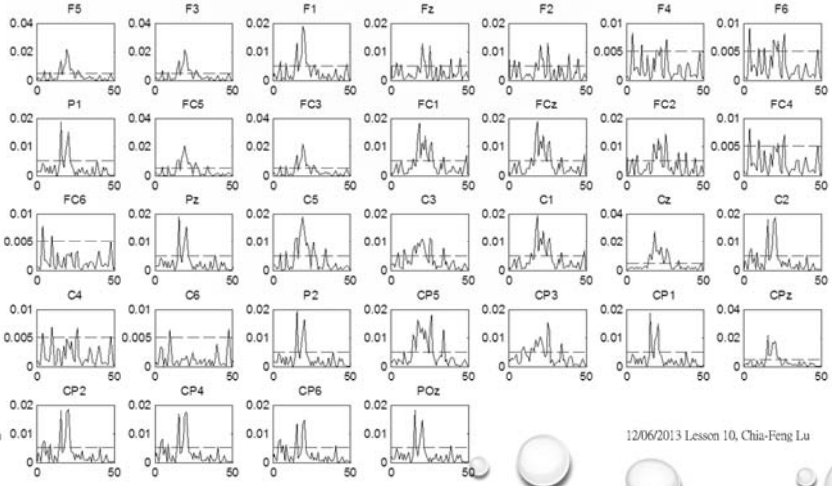
MVAR



腦電波/肌電訊號同調性分析應用

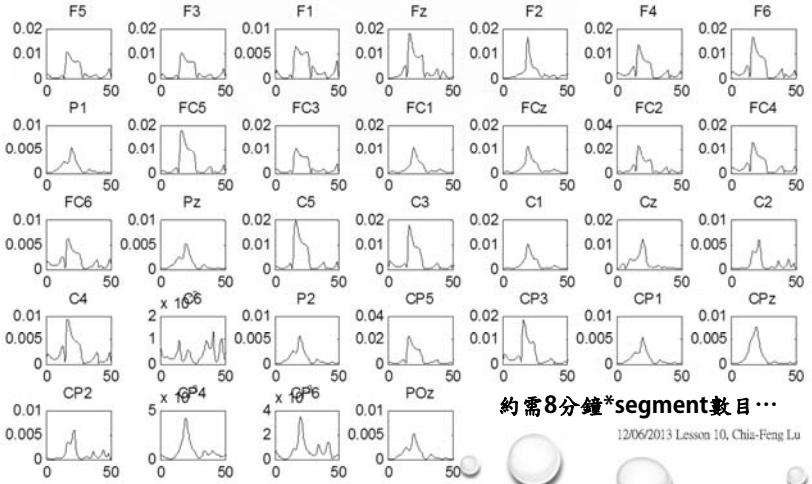
ANKLE DORSIFLEXION-右側TA連續收縮5分鐘

- 請開啟並執行demodata_L10\CONT\Cont_cohere.m



ANKLE DORSIFLEXION-右側TA連續收縮5分鐘

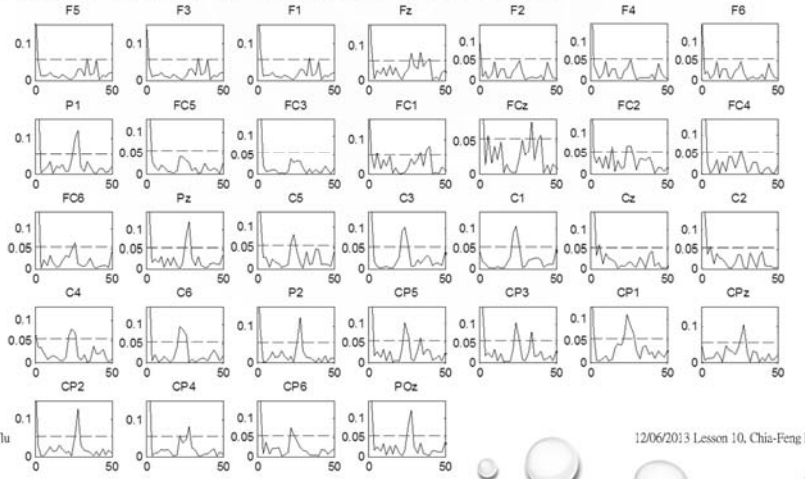
- demodata_L10\CONT\Cont_cohere_MVAR.m，但計算時間非常長!!



約需8分鐘*segment數目…

ANKLE DORSIFLEXION-單次收縮，54個TRIALS

- 請開啟並執行demodata_L10\TRIAL\Trial_cohere.m



THE END

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